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Misreporting Corporate Performance*

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Abstract

This paper develops a model of the causes and consequences of misreporting of corporate performance. We model why and when managers of public companies will choose to misreport and to invest in creating opportunities for misreporting. Even managers who cannot sell their shares in the short run might misreport in order to improve the terms upon which the company would be able to issue equity to finance new projects or stock acquisitions. When managers are free to sell some of their holdings in the short-run, incentives to misreport and the incidence of misreporting increase to an extent depending on the fraction of their holdings that managers may sell and on their ability to sell without the market knowing about it. Investments in misreporting have real economic costs and lead to distortions in capital raising decisions, with firms that misreport raising too much equity and firms that do not misreport raising too little. Lax accounting and legal rules increase investments in opportunities to misreport and the incidence of misreporting and, as a result, reduce ex ante share value. Our analysis provides a range of testable predictions concerning the periods, industries, and type of firms in which misreporting is likely to occur. The analysis also has implications for corporate governance and executive compensation.

JEL classification: G3, K22, M40

Keywords: Asymmetric information, acquisitions, corporate governance, disclosure, myopia, short-termism, executive compensation, stock options, insider trading, accounting, financial statements, earnings management, auditor, financial reporting.

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1. Introduction

Recent events have directed much public attention to the misreporting of corporate performance by publicly traded firms. Although some misreporting of corporate performance is a long-standing phenomenon,\(^1\) it is one with ebbs and flows, and the incidence of misreporting has substantially increased in the US in the second half of the nineties. Public attention has focused on “notorious” cases such as those of Worldcom, Tyco, or Enron, but the increase in the incidence of misreporting is a general phenomenon. While the number of earnings restatements by publicly traded firms was on average 49 per year from 1990 to 1997, such restatements numbered 91 in 1998, 150 in 1999, and 156 in 2000.\(^2\)

This paper investigates the causes and consequences of such misreporting. We develop a model of why and when misreporting occurs and the distortions associated with such misreporting. The model has positive implications concerning the circumstances in which misreporting is more and less likely to occur. The model also has implications for corporate governance. It identifies the efficiency costs of misreporting and how these costs depend on corporate governance arrangements.

Why and when would managers elect to misreport if cash flows are eventually going to be revealed so that the market cannot be ultimately fooled? To investigate this phenomenon, we study a five-stage model. First, managers decide how much to invest in creating opportunities to misreport in the event that they subsequently learn that the firm is of a low value type. Second, there is a stage in which managers learn information that suggests whether the expected value of the firm’s current projects is low or high. Managers are required (or at least may choose to) make statements to the market, and when doing so they might sometimes be able to misreport (the probability of managers being in a position to do so depends on their earlier investments in creating opportunities to misreport). Third, the learning of information and disclosure decisions stage is followed by market trading in the firm’s shares. Fourth, the firm operates and might have an opportunity to engage in a new project or acquisition that would require raising additional capital. In the fifth and final stage, payoffs from the

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\(^1\) The misreporting phenomenon seems to exits in different periods and in different countries. In the early 90s, questionable accounting practices led to corporate collapses in the UK. These scandals led to the introduction of more stringent accounting guidelines (See Smith (1992), The Economist (1992) and The Economist (1991)).

\(^2\) See Moriarty and Livingston (2001).
firm’s initial projects, and whatever additional project if any was added in the fourth stage, are realized and distributed.

Our model is one in which markets engage in rational pricing. Investors do not know what managers’ private information is, but they are aware of the possibility that managers will misreport, and they draw rational inferences from whatever they know of the managers’ actions as to what the managers’ likely information is.

We start with the case in which managers may not sell their own shares in the intermediate trading stage and must keep all of their shares until the final period. Even in this case, where managers must keep their shares until values are fully realized, and in which managers’ interests are aligned with those of “long-term” shareholders, managers might have an incentive to invest in creating opportunities to misreport and to misreporting when they can do so. When an opportunity to raise equity to finance an acquisition or a new project arises, misreporting enables the managers who learn that the firm’s current projects are of low value to pool themselves with firms with existing projects of high-value and in this way improve the terms upon which they would be able to raise capital. The initial shareholders (including the managers) of firms with existing projects whose value is lower than estimated by the market will be made better off if they will be able to obtain new capital on favorable terms, i.e., for a price exceeding what the outside investors are getting and the initial shareholders are giving up. This motive for misreporting might be especially important in circumstances in which firms are engaged in a series of stock-finance acquisitions. Worldcom and Tyco, for example, were “serial” acquirers.

Whereas misreporting might take place even when managers must keep their shares until the final realization period, the benefits to managers from misreporting, and the incidence of misreporting, increase in the case in which managers may sell some of their shares in the intermediate trading period. When managers who learn that their projects have a low expected value can and do misreport, the misreporting would enable them to gain by selling shares during the intermediate period. The selling of shares by managers of Enron enabled them to get out with substantial value before the market learned that the firm’s value was substantially lower than expected. The extent to which managers would gain from misreporting, and thus the increased incentive to do so, would depend on the fraction of the managers shares that they are permitted to sell. Thus, the greater the freedom of managers to sell their holdings in the short-run, the greater the incentive to misreport.
The extent to which managerial short-run selling can increase their gains from misreporting depends also on whether the market can tell whether such selling is taking place. The market’s ability to identify managerial selling in turn depends on whether and how frequently sales must be disclosed, the existence of trading windows, and the amount of shares managers might seek to sell relative to the ordinary volume of trading arising from liquidity selling. When the market knows or suspects that the managers are selling, the market price will decline to reflect an increased probability that the firm’s managers have been misreporting; this would reduce the profits that misreporting managers would be able to make by selling their shares in the intermediate trading period. However, as long as the market would not be able to tell whether known or suspected sales by managers are due to managers’ knowledge of negative information or due to managers’ liquidity needs, allowing managers to sell shares in the intermediate trading period will increase their gain from misreporting and their incentive to invest in creating opportunities to misreport.

One feature of our model is that the incidence of misreporting is endogenously determined and is a function of ex ante investments (such as in how operations and financial structures are set). These investments are influenced by various parameters of the firm, the industry, and the managers’ compensation package, which affect the potential benefits from misreporting. These investments are also influenced by the “technology” of misreporting, which is a function of legal and accounting rules (and the implementing institutional infrastructure) that are in place.

Our model has significant descriptive implications. The model can help shed light on the growth in misreporting in the second half of the 90’s. It identifies several factors that might have played a role in this growth. First, consider the many companies whose value was based primarily on future growth opportunities. For these companies, the difference between good and bad information about current operations (for example, about current revenues), which has little direct effect on the firm’s value, could still lead to a large difference in the market’s estimate of the firm’s future growth and business opportunities and thus of the expected future value of the firm’s projects. Second, many companies were engaged in stock-based acquisitions. Third, managers’ opportunities to sell large amount of shares in the short-run might have been substantial in that period. Specifically, the use of large stock option grants in compensation schemes became more common, and in the case of many high-tech firms, managers had some significant pre-IPO holdings which they could sell to the market. Fourth, because of reductions in the potential liability of auditors and
the structure of auditor services, auditors became more likely to acquiesce. As a result, the likelihood that a firm will be able to misreport (given its level of prior investment in creating opportunities to misreport) went up.

Our analysis provides a wide range of testable predictions concerning the circumstances - in terms of the period, the industry, the firm, and the managers’ payoffs -- in which misreporting is more and less likely to occur. Throughout, we identify relationships between the likelihood of misreporting and such parameters. Some of these relationships are consistent with already existing evidence, and others could be tested by future empirical work.

Our model also has corporate governance and policy implications. The analysis highlights that the phenomenon of misreporting does not have only distributive consequences but also gives rise to potentially significant efficiency costs. One type of cost arises from deadweight investments in creating opportunities to misreport and pool one’s firm with firms of higher value. Another important cost arises from distorting the allocative role of capital markets. When some low value firms can misreport and thereby pool themselves with high-value firms, the financing and investment decisions of both types of firms will be distorted. In the pooling equilibrium caused by misreporting, high-value firms will be cross-subsidizing those low-value firms whose managers will misreport. Because of this compelled cross-subsidization, high-value firms might forgo some efficient projects to avoid the need to raise capital, whereas some low-value firms that misreport might raise equity even when they do not have efficient projects. As a result, there will be under-investment by firms that do not misreport and over-investment by firms that do. The existence of such real economic costs can be expected to reduce ex ante share value.

Thus, arrangements that encourage misreporting can have a negative effect on share value. In our model, lax rules, which make it more likely that a firm will be able to misreport given its investment in creating opportunities to do so, are shown to increase the level of such investments and the incidence of misreporting. An important role of rules and institutions in our context is not simply to penalize some instances of misreporting ex post but also to affect the ex ante set of “misreporting opportunities.” Our analysis supports the calls by participants in public policy discussions (see, e.g., Levitt (1998)) for rigorous

\[ \text{Note:} \]

3 Coffee (2002) provides a compelling account of the legal and industry developments that led auditors to be more likely to go along with managers’ (mis)reporting decisions.

4 As will be discussed below (see remark 2 following proposition 3), these results are consistent with recent empirical evidence documenting the real effects of earnings manipulations (Polk & Sapienza (2002)).
accounting standards and implementing institutions, which can narrow firms’ “degrees of freedom” to engage in misreporting.

Our analysis also shows how the incidence of misreporting and its associated costs can be influenced by the design of corporate governance arrangements. We find that any increase in the fraction of managers’ holdings which they may sell before the “final period” will increase incentives to invest in creating opportunities to misreport and the incidence of misreporting. We also find that arrangements that eliminate or reduce the ability of managers to sell without the market’s knowledge – such as the recent legislative mandate that all trading by managers be very promptly disclosed – can reduce but not eliminate the adverse incentives created by managers’ freedom to unload holdings in the short-run. It is worth noting, however, that the incentives to invest in misreporting would decrease, but not be fully eliminated by arrangements that require or encourage managers to keep their shares for the long haul. Thus, while such arrangements might be helpful in addressing the problem of misreporting, their adoption would not make the problem (and thus the accounting and legal measures that can constrain it) irrelevant.

Our analysis is organized as follows. Section 2 presents our framework of analysis. Section 3 analyzes reporting and investment decisions in the case in which managers must keep their shares until the final realization period. Section 4 analyzes these decisions in the case in which managers are permitted to sell at least some of their shares before the final period. Section 5 discusses related literature, and Section 6 concludes.

2. Framework of Analysis

2.1 Sequence of Events

The sequence of events in the model is as follows:

T=0: Initial situation with (initially) identical publicly traded firms each run by a manager.

T=1: Managers may invest in creating opportunities for future misreporting of corporate performance.

T=2: Learning of information and disclosure decisions – managers learn information pertaining to the companies’ expected cash flows at the final period and make disclosure decisions.

T=3: Market trading.
T=4: Investment and financing decisions – the company may have an opportunity for a potentially beneficial project that would require raising additional capital.

T=5: Realization of payoffs.

We now specify the assumptions regarding each one of the six stages.

2.2 T=0: Initial Situation

At T=0, all companies are publicly traded, and each is run by a manager. Without loss of generality, we assume that at T=0 each company has one issued share that is held by initial shareholders including the company’s manager. The manager holds a fraction $\beta$ of the company’s stock. (The results would be essentially the same assuming that the manager has an option to purchase a fraction $\beta$ of the company’s stock.) The manager is assumed to be cash-constrained and thus cannot purchase additional equity. We shall abstract from other incentives that the manager might have (e.g., due to the threat of a control contest) and will assume that the manager will be making decisions for the firm in all of the model’s periods. We will further assume that the manager’s interest in enhanced share value comes solely from the specified holding of shares.

We initially assume that, due to legal or contractual constraints, managers may not sell shares at the T=3 market trading stage. Therefore, the manager’s objective is to maximize the price of the company’s stock at the final period. This assumption will be relaxed in Section 3, which will allow for managerial selling of shares in the intermediate trading period.

The T=0 value of a company, which is also the T=0 market price of the company’s share, is denoted by $P_0$. 

Fig. 1: Sequence of Events
2.3 T=1: Creating Opportunities for Misreporting

Investments in creating opportunities to misreport, and consequently the level of misreporting, are determined endogenously. At T=1 the manager invests \( C \in \mathbb{R}^+ \) in creating opportunities for future misreporting of unfavorable information concerning corporate performance.\(^5\) Specifically, given an investment \( C \) at T=1, if at T=2 the manager learns negative information, she will be able to misreport with probability \( \delta = \lambda \cdot \tilde{\delta}(C) \). We assume that \( \lambda \geq 0, \tilde{\delta}'(C) > 0, \tilde{\delta}''(C) < 0 \) and \( \tilde{\delta}'(0) \to \infty \).\(^6\)

The function \( \lambda \cdot \tilde{\delta}(C) \) represents the various legal and institutional factors in the economy that define the available “misreporting technology.” Many factors influence this technology, including legal rules, accounting standards and conventions, the structure of the accounting industry and accounting services (which in turn affect the ability of managers to influence auditors to go along), and the intensity of outside monitoring (by analysts, plaintiff lawyers, the financial press, etc.) A higher \( \lambda \) represents a more “lax” environment that makes it easier to misreport.\(^7\)

Theoretically, the investment \( C \) may create opportunities for misreporting of favorable information as well. Moreover, managers can perhaps invest \( C_g \) in creating additional opportunities for misreporting favorable information. However, it can be readily shown that managers will never misreport,

\(^5\) We assume that \( C \) is not observable to the market (although the market will be able to anticipate the level of \( C \) at equilibrium). Otherwise, managers would be able to effectively commit to truthful reporting simply by setting \( C = 0 \). Indeed, if \( C \) were observable and verifiable, we would expect shareholders (or those who took the firm public) to require managers (through corporate charters or contracts with the managers) not to invest in creation of opportunities to misreport.

\(^6\) An alternative assumption would be that \( C \) can be invested after the company’s type is revealed (at T=2). Similar results will hold under this alternative assumption. Fischer and Verrecchia (2000) study a model where misreporting is costly (they study a single-period model, so the question when the cost of distorting the report is incurred does not come up).

\(^7\) We shall, for simplicity of exposition, assume that the level of \( \lambda \) is given by the environment and is not a parameter chosen by the firm itself. Of course, although the environment clearly plays a key role in shaping the scope of misreporting opportunities, companies might be able to adopt observable arrangements that influence the level of \( \lambda \). Our model can be easily extended to the case in which the environment defines a range of values for \( \lambda \) from which firms can choose. In our model, if companies could lower \( \lambda \) in an observable fashion, it would be ex ante optimal for shareholders to do so. Accordingly, one can simply interpret the analysis below assuming that the \( \lambda \) it uses is the lowest possible given the legal and institutional environment.
announcing that their information is negative, when in fact it is positive. It follows that managers will not have any reason to invest in creating opportunities to hide favorable information. Therefore, without loss of generality, we assume that the investment \( C \) creates only opportunities to misreport unfavorable information.

### 2.4 T=2: Learning of Information and Disclosure Decisions

At \( T=2 \) the manager of each company learns some information pertaining to the ultimate \( T=5 \) value of the company’s existing project. The information may be either “good” or “bad”, i.e. \( i \in \{g, b\} \). The manager receives bad information with probability \( \theta \), and good information with probability \( 1 - \theta \). The expected value of the company’s \( T=5 \) cash flows conditional on the \( T=2 \) information is denoted by \( E(V|I) \). Specifically, \( V_H = E(V|g) \) denotes the expected value of the final \( (T=5) \) payoff from the firm’s existing assets given good information, and \( V_L = E(V|b) \) denotes the expected value of this final payoff given bad information.

We assume that \( V_H > V_L \), and denote the difference between the two expected values as \( \Delta V = V_H - V_L \).

The information that the manager learns at \( T=2 \) may be any information that is relevant for estimating the expected final value to be produced by the firm’s existing projects. The information might concern current revenues, costs, or earnings. It might concern the company’s volume of activity, the establishment of strategic or other relations, and so forth. For concreteness, we shall speak below about the \( T=2 \) information as pertaining to the company’s earnings, but it will be clear that the results apply to any other type of information. Specifically, we shall assume that the manager may learn good information indicating that current earnings are high, \( E_H \), or bad information indicating that current earnings are low, \( E_L \). Let \( \Delta E = E_H - E_L \) denote the difference between low and high earnings.

Knowing the current level of earnings affects the estimate of the expected final value of the company’s existing assets. A company with low earnings and thus a low expected final value, \( V_L \), will be called a type L company, and a company with high earnings and thus a high expected final value, \( V_H \), will be termed a type H company. The ratio \( \mu \equiv \frac{\Delta V}{\Delta E} \) represents the multiple by which

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8 As demonstrated below, both high value companies and low value companies will invest \( C \) at \( T=1 \). The expected values \( V_L \) and \( V_H \) already incorporate the investment \( C \).
the difference between the two possible levels of current earnings affects the expected final value.

Managers are required to state an earnings figure (e.g., in the company’s financial statements), and they announce either $E_H$ or $E_L$. As determined by the T=1 investment, there is a probability $\delta$ that the manager of a company with low earnings will be able to misreport and announce high earnings. Our analysis can also be alternatively interpreted as applying to a situation in which disclosure is voluntary. In such case, managers will announce $E_H$ or say nothing, and managers with low earnings will be able with probability $\delta$ to misreport and announce high earning $E_H$ if they so choose. The discussion below will use the mandatory disclosure model, but we will on occasion note the particular voluntary disclosure interpretation of our results.

Whereas at T=2 the manager learns the company’s current earnings, the market does not directly observe these earnings. Investors only know the probability $\theta$ of low earnings, and they make whatever inference can be rationally drawn from the manager’s announcement. The T=2 market price of the company’s share will be denoted by $P^2$.

2.5 T=3: Market Trading

At T=3 market trading occurs, because some shareholders must sell for liquidity reasons, and a price $P^3$ is set for the company’s stock. Given our current assumption that legal or contractual restrictions prevent the manager from trading at T=3, the T=3 price cannot reflect any new information, i.e., $P^3 = P^2$.

2.6 T=4: Investment and Financing Decisions

At T=4, the manager might learn -- with probability $\gamma$ -- of a potential new and profitable project. This project requires an investment $K$. Our analysis can be viewed as covering both the case in which this capital is needed to build the new project from scratch and the case in which this capital is needed to acquire another company. The project will increase the final cash flow by $K + R$, where $R$ is distributed over $\mathbb{R}^+ = [0, \infty)$ with a positive and continuous pdf $f(R)$ and a cdf $F(R)$. Let $\bar{R} = E(R)$ denote the average return of the new project. Note that for

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9 A manager can only hide information *temporarily* by skewing the company’s financial statements. At T=5, cash flows are observed by the market, subject to noise (see section 2.7 infra).
now we are assuming that the new project, if it emerges, would be an efficient one, with the uncertainty being only about its profitability, but we shall drop this assumption and allow for inefficient projects in Section 3.6.

The manager knows \( R \) at \( T=4 \). The market, however, knows only the distribution \( f(R) \). In the event that the company elects to raise capital, the market will also make whatever inferences can be drawn from the manager’s \( T=2 \) announcement and the \( T=4 \) decision to raise capital. The \( T=4 \) market price of the company’s stock is denoted by \( P^4 \).

We shall for now assume that capital can be raised only by issuing equity; Section 3.7 will consider the case of issuing debt. In the case of financing an acquisition, the new equity might be given directly to the shareholders of the acquired company or it might be sold to third parties and the cash obtained form them given to the selling shareholders. We shall denote by \( \hat{\alpha} \) the number of new shares that would have to be sold to raise \( K \) through issuing new equity. Selling \( \hat{\alpha} \) shares would involve giving up a fraction \( \alpha = \frac{\hat{\alpha}}{1 + \hat{\alpha}} \) of the company’s \( T=5 \) total cash flows, and it will be convenient to use \( \alpha \), rather than \( \hat{\alpha} \), in the mathematical derivations. Let \( \alpha(E_H) \) and \( \alpha(E_L) \) denote the fractions of the company that will need to be sold in order to raise \( K \) when managers announce \( E_H \) and \( E_L \), respectively.

### 2.7 T=5: Realization of Payoffs

At \( T=5 \), all cash flows are realized. The company’s initial project will produce cash flows of \( V + \varepsilon_o \), where \( V \in \{V_L, V_H\} \) and \( \varepsilon_o \) is a random zero-mean noise. The company’s new project, if one was undertaken at \( T=4 \), will produce cash flows of \( K + R + \varepsilon_N \), where \( \varepsilon_N \) is a random zero-mean noise. If at \( T=1 \) the company invested in creating opportunities to misreport its earnings at \( T=2 \), cash flows are reduced by the cost \( C \) of doing so. The final \( T=5 \) stock price is denoted by \( P^f \).

Note that the presence of noise implies that it is not possible to infer clearly from a company’s \( T=5 \) cash flows whether or not misreporting took place at \( T=2 \). When a company reported high earnings at \( T=2 \), a relatively low value at \( T=5 \) could be due to an unfavorable realization of uncertainty rather than to misreporting at \( T=2 \). Of course, while the model assumes that whether misreporting occurred is not directly observable, in reality ex post investigations sometimes unearth evidence that misreporting took place. We shall assume for simplicity that no ex post penalties will be imposed at \( T=5 \). Our model, however,
can be easily extended to the case in which misreporting is penalized ex post with some probability. In such a case, misreporting will take place only if the benefits from it, as derived below, exceed the expected sanction. The results presented below are qualitatively robust to such an extension (adding this threshold condition for misreporting).

3. Reporting and Investment Decisions

As is conventional, we solve the model by backward induction, starting with the T=4 financing and investment decisions. We first examine how decisions at this stage would be made if no misreporting took place earlier. We then study how these decisions would be made in the presence of misreporting.

3.1 Financing and Investment Decisions Without Misreporting

Consider first the benchmark case in which no company can misreport and thus all companies issue truthful statements at T=2. In this case, since \( R \geq 0 \), all companies that face a new project will issue equity and raise \( K \) to fund it. Assuming truthful statements, let \( \alpha_H = \alpha(E_H) \) and \( \alpha_L = \alpha(E_L) \) denote the fractions of the company that need to be sold in order to raise \( K \) when managers announce \( E_H \) and \( E_L \), respectively. Specifically, in order to raise \( K \):

- H companies will sell a fraction \( \alpha_H \) of the company such that \( \alpha_H \cdot (V_H + K + R) = K \); and
- L companies will sell a fraction \( \alpha_L \) of the company such that \( \alpha_L \cdot (V_L + K + R) = K \).

Clearly, \( \alpha_L > \alpha_H \), i.e. L companies will have to sell a larger fraction of their T=5 cash flows in order to fund the new project.

In the truthful statements case, the expected final T=5 per-share prices for H and L companies are \( E(P_H^t) = (1 - \alpha_H) \cdot (V_H + K + R) \) and \( E(P_L^t) = (1 - \alpha_L) \cdot (V_L + K + R) \), respectively.\(^{10}\) In this model, these will be the manager’s (per-share) payoffs, depending on her company’s type. Therefore, the manager will always sell

\(^{10}\) The expected T=5 cash flows of the company are \( V_H + K + R \) for H companies and \( V_L + K + R \) for L companies. To get the per-share market price, we divide these values by the number of outstanding shares, \( 1 + \hat{\alpha} \). Recall that \( \alpha = \hat{\alpha}/(1 + \hat{\alpha}) \), which implies \( 1 + \hat{\alpha} = 1/(1 - \alpha) \).
equity to finance the new project. A manager facing a below average new project, i.e. \( R < \bar{R} \), will clearly sell equity to finance this new project. This manager enjoys both the positive revenues from the new project and a cross-subsidization effect that lowers the number of shares that must be sold to raise \( K \). A manager facing an above average new project, i.e. \( R > \bar{R} \), will also sell equity to finance the new project, since the high revenues more than offset the cross-subsidization effect.\(^{11}\)

### 3.2 Financing and Investment Decisions with Misreporting

In our model, some L firms might have an opportunity to misreport at \( T=2 \). When L companies with an opportunity to misreport mimic H companies and announce \( E_H \), the market cannot distinguish between these two types of companies. Consequently, a single “pooling” price is set for all the companies that announce \( E_H \). Let \( \alpha \equiv \alpha(E_H) \) denote the fraction of the \( T=5 \) cash flows that a company that announces \( E_H \) will have to sell in order to fund a new project when L companies misreport earnings.

Specifically, in order to raise \( K \), managers must sell a fraction \( \alpha \) of the company such that \( \alpha \cdot \Pi = K \), where \( \Pi \) is the expected value of a company conditional on the fact that the company reported \( E_H \) and is selling equity. This expected value is given by:

\[
\Pi = V_H - \left( \theta \cdot \delta \cdot \Pr(L \text{ sell})/\rho \right) \cdot \Delta V + K + \left[ \theta \cdot \delta \cdot \Pr(L \text{ sell}) \cdot \bar{R}_L + (1 - \theta) \cdot \Pr(H \text{ sell}) \cdot \bar{R}_H \right] / \rho,
\]

where \( \Pr(L \text{ sell}) \) is the probability that an L company that misreports and has a new project sells equity, \( \Pr(H \text{ sell}) \) is the probability that an H company that has a new project sells equity, \( \rho = \theta \cdot \delta \cdot \Pr(L \text{ sell}) + (1 - \theta) \cdot \Pr(H \text{ sell}) \) is the overall probability that a company that has a project and reported \( E_H \) sells equity, \( \bar{R}_L \) is the expected value of the profit from an L company’s new project conditional on the company announcing \( E_H \) and selling equity, and \( \bar{R}_H \) is the expected value of the profit from an H company’s new project conditional on the company selling equity.

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\(^{11}\) Formally, the manager will sell equity if and only if \((1 - \alpha) \cdot (V + K + R) > V\), where \( \alpha \) satisfies \( \alpha \cdot (V + K + \bar{R}) = K \). This condition can be rewritten (after some rearranging) as \( R > -K \cdot \bar{R} / (V + \bar{R}) \), which implies that the manager will sell equity for all \( R \).
To proceed, we need to derive the probabilities $\Pr(L\ sell)$ and $\Pr(H\ sell)$ and to identify the circumstances in which each type of company sells equity. This is done in the following proposition.

**Proposition 1:** When $L$ companies that can misreport do so at $T=2$, then, in the event that a profitable new project emerges at $T=4$ -

(i) Managers of $L$ companies, both those that misreported and those that did not, will always sell equity to fund the project.

(ii) Managers of $H$ companies will sell equity if and only if the profitability of the new project exceeds a threshold $\hat{R}_H$, which is defined by the following equation:

$$
\theta \cdot \delta \cdot \hat{R} + (1 - \theta) \cdot \int_{\hat{R}_H}^{\infty} R \cdot f(R) \cdot dR - \theta \cdot \delta \cdot \Delta V \\
\theta \cdot \delta + (1 - \theta) \cdot \left(1 - F(\hat{R}_H)\right)
$$

Remark: The intuition for this result, whose proof is provided in the Appendix, is as follows:

In the truthful statements case, when there is no misreporting at $T=2$, we have seen that managers will always sell equity to finance new projects. Introducing misreporting by $L$ companies adds a cross-subsidization effect, which stems from the pooling between $H$ companies and $L$ companies that misreport earnings. This cross-subsidization will make misreporting $L$ companies all the more eager to sell equity in order to finance their new projects, because they will now need to sell fewer shares at $T=4$ (compared with the case in which there is no misreporting).

The same cross-subsidization effect might prevent $H$ companies from pursuing new projects, since they will now need to sell more shares at $T=4$ in order to raise $K$. Specifically, $H$ companies facing a profitable, yet insufficiently attractive project, i.e. a new project with $R < \hat{R}_H$, will forgo the new project.

From proposition 1, we know that $L$ companies that misreport earnings will always sell equity, i.e. $Pr(L\ sell) = 1$. Therefore, $\rho = \theta \cdot \delta + (1 - \theta) \cdot Pr(H\ sell)$. Since the proportion of $L$ companies among companies that announce $E_H$ and sell equity is of central importance, we define $\omega \equiv \theta \cdot \delta / \rho$. Also, if all $H$ companies sell equity, i.e. if $\hat{R}_H = 0$ and $Pr(H\ sell) = 1$, we define $\omega_0 = \frac{\theta \cdot \delta}{\theta \cdot \delta + (1 - \theta)}$. Using these definitions, we can state the following Corollary.

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Corollary 1 (Efficiency Costs): As long as the difference between L and H types satisfies \( \Delta V > \frac{R}{\omega_0} \), then -

(i) Some H Companies will not finance and invest in new efficient projects.

(ii) The likelihood that an H company will not finance and invest in a new efficient project that it faces is increasing in the threshold value \( \hat{R}_H \), which in turn is an increasing function of \( \Delta V \) and \( \mu : \frac{\partial \hat{R}_H}{\partial (\Delta V)} > 0 \) and \( \frac{\partial \hat{R}_H}{\partial \mu} > 0 \).

(iii) Among companies that announce \( E_H \), companies that misreport earnings will be more likely to raise capital.

Remark 1 (intuition): The intuition for this result, whose proof is provided in the Appendix, is as follows:

(i) In the symmetric information case, absent cross-subsidization of L companies, H companies will always sell equity to pursue the new project if it emerges. Misreporting introduces the cross-subsidization effect, which imposes an additional cost, \( \omega_0 \cdot \Delta V \), on H companies that sell equity. When this cross-subsidization cost is sufficiently small (\( \omega_0 \cdot \Delta V < \hat{R}_H \)), then H companies will always sell equity even in the presence of misreporting. However, when the cross-subsidization effect is significant (specifically, if \( \omega_0 \cdot \Delta V > \hat{R}_H \)), H companies will sell equity only when facing a new project that is sufficiently profitable, namely when \( R > \hat{R}_H \).

(ii) When the cross-subsidization effect is significant, i.e. when \( \omega_0 \cdot \Delta V > \hat{R}_H \), H companies will forgo efficient projects with \( R < \hat{R}_H \). Therefore, the likelihood that an H company will not finance and invest in new efficient projects is increasing in \( \hat{R}_H \).

The threshold value, \( \hat{R}_H \), depends on the magnitude of the cross-subsidization loss that H companies must bear if they choose to sell equity, as measured by \( \omega_0 \cdot \Delta V \). When the difference between the expected value of the initial projects of H and L companies, \( \Delta V \), is larger, the cross-subsidization effect is also larger. Put differently, since \( \Delta V = \mu \cdot \Delta E \), when the impact of misreported earning on the estimate of the T=5 final value (as measured by \( \mu \)) is greater, the cross-subsidization effect is larger.

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For this part of the corollary it is necessary to assume that \( f(\hat{R}_H) \) is not too large (the precise condition is provided in the proof in the Appendix).
Note that, when $\omega_0 \cdot \Delta V > \bar{R}$ and $\hat{R}_H > 0$, H companies might not raise capital to fund efficient projects. Specifically they will not undertake projects with $R \in (0, \hat{R}_H)$. Therefore, The possibility of misreporting might lead to allocative inefficiency, generating a real economic cost.

(iii) When $\omega_0 \cdot \Delta V > \bar{R}$, each L company that misreports and announces $E_H$ will sell equity. On the other hand, among the H companies, which all also announce $E_H$, only companies facing a sufficiently profitable new project, with $R > \hat{R}_H$, will sell equity. Hence, among companies that announce $E_H$, those that misreport earnings are subsequently more likely to sell equity.

**Remark 2 (empirical implications):** Corollary 1 provides us with the following testable predictions for future empirical work:

(i) Companies that restate earnings, or are otherwise found to have misreported, are more likely to have subsequently sold equity. This prediction is consistent with recent empirical evidence documented by Lang and Lundholm (2000).\(^{13}\)

(ii) The greater the magnitude of the misreporting of earnings, or the more significant the misreporting in terms of its implication for the expected final value, the more likely it is to be followed by an equity sale.\(^{14}\)

### 3.3 The Reporting Decision

After analyzing the T=4 financing and investment decisions, we now move one step backwards in time, and solve for the T=2 decision of L companies whether to misreport earnings.\(^{15}\)

**Proposition 2:** In the unique equilibrium, all L companies that can misreport at T=2 will elect to do so.

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\(^{13}\) Palmrose & Scholz (2000) and Palmrose, Richardson & Scholz (2001) collect data on restatements by companies. This type of data can be used to test the predictions derived from our theoretical model.

\(^{14}\) When $\Delta V$ (or $\mu$) are higher, the threshold $\hat{R}_H$ is higher, which means that fewer H companies sell equity. Since L companies always sell equity, if fewer H companies sell equity, then from the pool of companies that announce $E_H$ and sell equity, the share of misreporting L companies increases. Consequently, the correlation between misreporting and selling equity increases.

\(^{15}\) Since we are currently assuming that managers cannot sell stock at T=3, we can skip period 3.
Remark 1 (intuition): The intuition for this result, whose detailed proof is omitted, is as follows:

An L company clearly gains from announcing \( E_H \) (rather than the true \( E_L \)). Whatever the other companies report, if an L company reports \( E_H \), the market will assign a larger probability that the company is of type H, as compared to the case in which the company announces \( E_L \). Hence, misreporting will reduce the cost of raising capital. And, since misreporting is costless at T=2 (the cost \( C \) of creating opportunities to misreport is sunk at this stage), misreporting would be a dominant strategy for any L company that can misreport.

Remark 2 (the gain from misreporting): As noted above, the gain from misreporting derives from the more favorable terms for raising equity – i.e., from having to sell fewer shares to finance the new project. In particular, without misreporting an L company will have to sell a fraction \( \alpha_L \) of the company such that \( \alpha_L \cdot (V_L + K + R) = K \), leaving it with an expected value of \((1-\alpha_L) \cdot (V_L + K + R) \). In contrast, an L company that misreports will need to sell only a fraction \( \alpha_p \) of the company, defined by \( \alpha_p \cdot \Pi = K \), leaving the initial shareholders with an expected value of \((1-\alpha_p) \cdot (V_L + K + R) \). Therefore, recalling that a new project will emerge with probability \( \gamma \), the gain from misreporting is

\[
G = \gamma \cdot (\alpha_L - \alpha_p) \cdot (V_L + K + R).
\]

The gain from misreporting is increasing in the probability that a new project will become available, \( \gamma \). Also, the gain from misreporting is decreasing in \( \alpha_p \), or equivalently is increasing in \( \Pi \), the expected value of a company that announces \( E_H \) and sells equity. Since \( \Pi \) is decreasing in \( \delta \), the gain from misreporting is also decreasing in \( \delta \). Intuitively, when the level of misreporting is higher, the market will know that among companies announcing \( E_H \) there are more L companies. Consequently, the market will offer a lower price per-share for companies that announces \( E_H \), reducing the gain from misreporting.

Finally, the gain from misreporting is increasing in \( \Delta V \) (or \( \mu \)). When the difference in value between L companies and H companies is greater, L companies have more to gain from pooling with H companies.
3.4 Creating Opportunities to Misreport

At T=1 managers decide how much to invest in creating opportunities to misreport earnings. The equilibrium level of this investment decision is characterized in the following proposition.

Proposition 3: In the unique equilibrium, all companies invest \( C^* \) at T=1 in creating opportunities to misreport, where \( C^* \) is defined by \( \lambda \cdot \tilde{\delta}'(C^*) \cdot G(\delta(C^*)) = 1 \). The level of investment, \( C^* \), and as a result the overall level of misreporting, \( \delta(C^*) \), is:

(i) increasing in the laxity of the legal and accounting environment, as measured by \( \lambda \);
(ii) increasing in the probability that a company will face a new project, \( \gamma \);
(iii) increasing in the magnitude of the difference in value between H and L companies, \( \Delta V \), and thus in the significance of the misreporting for the expected final value, as measured by \( \mu \).

Remark 1 (intuition): The intuition for this result, whose proof is provided in the Appendix, is as follows:

A single manager has no influence on the overall level of misreporting, and therefore takes \( \delta(C^*) \) as given in her T=1 decision concerning how much to invest in creating opportunities to misreport. The manager therefore will increase \( C \) as long as the marginal benefit of \( \lambda \cdot \tilde{\delta}'(C) \cdot G(\delta(C^*)) \) exceeds the marginal cost:

(i) When the legal and accounting environment is more lax, the marginal benefit of investment in creating opportunities – in terms of the increased probability of being able to misreport – is larger. Consequently, managers will invest more in creating opportunities to misreport. This result implies that, by reducing the laxity of the legal and accounting environment, we may be able to lower investment in creating opportunities to misreport and in turn the incidence of misreporting.

\[ \text{Given } \delta(C^*), \text{ there is a unique investment level, } C, \text{ that satisfies the FOC, } \lambda \cdot \tilde{\delta}'(C) \cdot G(\delta(C^*)) = 1. \text{ At equilibrium, the many individual managerial choices of } C, \text{ and consequently of } \delta(C) \text{ must induce the aggregate level of misreporting, } \delta(C^*). \text{ Hence the condition: } \lambda \cdot \tilde{\delta}'(C^*) \cdot G(\delta(C^*)) = 1. \text{ Since both } \delta(C) \text{ and } G(\delta(C)) \text{ are decreasing in } C, \text{ the condition } \lambda \cdot \tilde{\delta}'(C^*) \cdot G(\delta(C^*)) = 1 \text{ defines a unique level of investment in creating opportunities to misreport.} \]
(ii) As explained in section 3.3, the gain from misreporting is increasing in the probability that a new project will emerge. Therefore, when the probability $\gamma$ increases, managers will invest more in creating opportunities to misreport.

(iii) As explained in section 3.3, the gain from misreporting is increasing in the difference between the value of H and L companies, $\Delta V$, i.e., in the significance of the misreporting for the estimated expected final value, $\mu$. Therefore, when $\Delta V$ and $\mu$ are higher, managers will invest more in creating opportunities to misreport.

 Remark 2 (empirical implications): The results stated in proposition 3 provide the following testable predictions:

(i) The result stated in part (i) of proposition 3 is consistent with empirical evidence regarding the positive effects of switching to more strict accounting standards (see Leuz and Verrecchia (2000)).\(^{17}\) It is also consistent with evidence indicating that managers take advantage of minimal disclosure requirements to engage in earnings management (see Lobo & Zhou (2001)).

Furthermore, since the effectiveness of any set of legal or accounting standards and practices varies across industries, proposition 3(i) suggests cross-sectional variations in the level of misreporting. This result is consistent with recent empirical evidence documenting more severe real effects of earnings manipulation in R&D intensive companies where existing accounting standards provide for only limited transparency (see Polk & Sapienza (2002) and Aboody and Lev (2000)).

(ii) The result stated in part (ii) of proposition 3 suggests the following testable predictions:

(a) Cross-sectionally, in industries where companies are likely to face new opportunities that require additional capital, misreporting of earnings is more likely to occur.

(b) Comparing different time periods, in periods when more companies face such new opportunities, misreporting of earnings is more likely to occur. Relatedly, in periods when there are many equity offerings, misreporting of earnings is more likely to occur.

(iii) The result stated in part (iii) of proposition 3 provides the following testable predictions:

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\(^{17}\)Leuz and Verrecchia (2000) document the lower bid-ask spreads and higher trade volumes enjoyed by German firms that switched from the German reporting regime to an international reporting regime (IAS or U.S. GAAP).
(a) Cross-sectionally, in industries where the impact of misreported earnings on the estimated final value (as measured by $\mu$) is greater, misreporting is more likely to occur. This prediction is consistent with the evidence that information asymmetries are especially large in R&D intensive industries (see Aboody and Lev (2000)). In such industries, misreporting is more likely to occur.

(b) Comparing different time periods, in periods when managerial misreporting has a large impact (as measured by $\mu$) on the estimated value of the initial projects, misreporting is more likely to occur.

3.5 The Ex Ante Cost of Misreporting

We can now state the magnitude of the efficiency cost generated by misreporting.

**Proposition 4:** Misreporting generates an expected efficiency cost of:

$$\Phi = C^* + \gamma \cdot (1 - \theta) \cdot \Pr(R < \hat{R}_H) \cdot E(R|R < \hat{R}_H).$$

Therefore, with misreporting, the ex ante $T=0$ value is reduced by $\Phi$ to $V_0 = V_H - \theta \cdot \Delta V + \gamma \cdot \bar{R} - \Phi$. The efficiency cost, $\Phi$, and thus the reduction in ex ante value, are:

(i) increasing in the laxity of the legal and accounting environment, as measured by $\lambda$;

(ii) increasing in the probability that a company will face a new project, $\gamma$;

(iii) increasing in the magnitude of the difference between L and H companies, $\Delta V$, and thus in the significance of the misreporting for the expected final value, as measured by $\mu$.

**Remark 1 (intuition):** The intuition for this result, whose detailed proof is omitted, is as follows:

Misreporting leads to two types of efficiency costs, which are reflected in the two elements of $\Phi$:

(1) The deadweight cost of creating opportunities to misreport, $C^*$.

(2) Inefficient investment decisions: With probability $\gamma$ the company faces a new efficient project. However, if this company enjoyed high earnings ($E_H$ occurs with probability $1 - \theta$), and the new project is not sufficiently profitable ($\Pr(R < \hat{R}_H)$), then this efficient project will not be pursued, leading to an expected loss of $E(R|R < \hat{R}_H)$.
As stated in proposition 3, an increase in the laxity of the legal and accounting environment, in the probability that a company will face a new project, in the magnitude of misreporting and in the significance of the misreporting for the expected final value – all lead to a higher investment in creating opportunities to misreport. These factors thus increase both the first element of $\Phi$ and the second element of $\Phi$. When the level of misreporting is higher, H companies will be more reluctant to raise capital in order to finance a new project. These companies will invest in the new project only if its returns are especially high. Formally, a higher level of misreporting increases the threshold $\hat{R}_{\text{II}}$, and with it the fraction of projects that will be forgone and the expected loss from forgoing such projects.

**Remark 2 (empirical implications):** The results stated in proposition 4 provide the following testable predictions:

(i) Companies located in countries with a legal and accounting environment that constrains misreporting to a greater extent will enjoy a higher Tobin’s Q.\(^{18}\) Similarly, changes in the legal and accounting environment that make it more difficult to create opportunities to misreport will increase share value.

(ii) To the extent that companies can through private action reduce the level of the parameter $\lambda$ in an observable way, such reductions would raise ex ante share value. This conclusion is consistent with the results in Fishman and Hagerty (1989, 1990) and Dye (1990) (see also Verrecchia (2001, sec. 4) and the references he cites) that commitments to improved disclosure raise ex ante share value.

### 3.6 Inefficient Projects

We have shown that the possibility of misreporting has real economic costs. In particular, H companies might not pursue efficient projects. In a more general model, misreporting leads to a second efficiency cost – that L companies might pursue inefficient projects. To show this point, we now extend the basic model to allow for inefficient projects.

In particular, we introduce the possibility of inefficient projects with $R = -\tilde{R}$ ($\tilde{R} > 0$). We assume that, in the event that a new project emerges, the project will be inefficient (i.e. $R = -\tilde{R}$) with probability $p$, and with probability $1 - p$ the project will be efficient, with $R$ being distributed as before over $\mathbb{R}^+$ according to

---

\(^{18}\) See Leuz and Verrecchia (2000) for some preliminary evidence consistent with this prediction.
the pdf $f(\cdot)$. The equilibrium in this extension, which allows for inefficient projects, is as follows.

**Proposition 5:** In the case in which the new project might be inefficient, if $\Delta V$ is sufficiently large, or equivalently if $\mu$ is sufficiently large, then -

(i) L companies that misreport and announce $E_H$ will invest in inefficient projects.

(ii) $H$ companies will pursue fewer efficient projects, as compared to the case in which all new projects are efficient. The threshold for investing is characterized by $\frac{\partial \tilde{R}_H}{\partial p} > 0$ and $\frac{\partial \tilde{R}_H}{\partial R} > 0$.

**Remark 1 (intuition):** The intuition for this result, whose proof is provided in the Appendix, is as follows:

(i) When facing an inefficient project, managers of L companies that misreport might still choose to sell equity. While the initial shareholders of the L company (including the manager) will bear part of the loss produced by the inefficient project, they also will gain from the cross-subsidization effect involved by raising equity while pooling with H companies. When the latter effect dominates, managers of L companies that misreport will sell equity even when facing an inefficient project. As $\Delta V$ increases, the cross-subsidization gain increases as well, making an equity offering more appealing.

(ii) When L companies that misreport pursue inefficient projects, the magnitude of the cross-subsidies effect increases. Consequently, H companies will be more reluctant to sell equity, i.e., will sell equity only when the profitability of the new project exceeds a higher threshold. The magnitude of the cross-subsidization effect is increasing in the fraction of projects that are inefficient, $p$, and in the magnitude of the inefficiency of these projects, $\tilde{R}$. Consequently, a higher $p$ and a higher $\tilde{R}$ decrease the fraction of efficient projects that H companies will choose to undertake.

(iii) Note that the possibility of inefficient new projects introduces a form of inefficiency that was absent from the basic model. When the difference between the value of the initial projects of H and L companies, $\Delta V$, is sufficiently large, and thus the cross-subsidization effect is sufficiently large, L companies will pursue inefficient projects. Equivalently, when the impact of managerial misreporting on the estimated final value of the firm’s initial projects is sufficiently large (i.e., if $\mu$ is sufficiently large), the cross-subsidization effect will be sufficiently large, and L companies will pursue inefficient projects. Thus, once
inefficient projects are introduced into the model, misreporting generates yet another form of allocative inefficiency resulting from the undertaking of inefficient projects.

**Remark 2 (testable predictions):** The result stated in proposition 5 implies that inefficient projects – projects that destroy rather than enhance value -- will more likely be undertaken by companies that misreported prior to undertaking the project. Recent evidence on the poor long-term performance of companies that engage in earnings management – either at the IPO stage (Teoh et al. (1998a)) or prior to a seasoned equity offering (Teoh et al. (1998b)) - is consistent with this prediction.

### 3.7 Debt Financing

We have thus far assumed that in order to fund a new project, companies will use equity financing. However, the possibility of misreporting raises similar problems also when companies use debt financing (which both Enron and WorldCom, for example, substantially did). As long as companies face a positive probability of insolvency, the interest rate on a company’s debt will be determined by the market’s beliefs regarding the company’s type (H or L). Consequently, misreporting will affect the interest rate, i.e. the price of debt. In the same way that L firms that misreport will be able to raise equity on more favorable terms than L firms that do not misreport, L firms that misreport will also be able to raise debt on more favorable terms than L firms that do not misreport.

The analysis of the debt-financing model can proceed in the same way that we have pursued above with similar conclusions. As before, investment in creating opportunities to misreport, and the level of misreporting, are increasing in the laxity of the legal and accounting environment, in the probability that the company will face a new project, and in the potential significance of the misreporting for the estimated expected final value. L companies that misreport will enjoy a cross-subsidization gain from pooling with the H companies, whereas H companies that will raise capital will be hurt by this cross-subsidization effect and will have to pay higher interest rates. Consequently, H companies will forgo some efficient new projects.
4. Managerial Selling of Shares before the Final Period

The setup presented in Section 2 and analyzed in Section 3 assumed that managers may not sell shares until the final period, and they thus focus on maximizing the long-term (T=5) value of the company’s stock. This assumption will hold when, for example, compensation and employment contracts preclude managers from selling shares in the intermediate trading stage (T=3). However, managers commonly may sell at least some of their holdings in the short-run. In this section, we therefore explore the implications of the possibility that managers will sell shares in the intermediate trading period. We show that this option increases the incentives of managers of L companies to invest in creating opportunities to misreport and thus increases the incidence of misreporting and the efficiency costs generated by it.

4.1 Introducing Managerial Selling

We now allow managers at T=3 to sell shares at the per-share price $P^3$. We assume that the manager may sell (up to) $\beta \in [0, \beta]$ shares, where $\beta$ is determined by the manager’s executive compensation contract. Two reasons are usually given as to why executive compensation contracts permit selling of shares prior to the final period. First, a manager might experience a liquidity shock. Second, managers are often risk-averse, and thus will suffer disutility from bearing the risk involved in the realization of the final period noise. Without loss of generality, we focus in the analysis below on the liquidity shock reason (Fishman and Hagerty (1995) adopt a similar assumption).

In particular, we assume that with probability $q$ managers experience a liquidity shock which leads them to wish to sell their holdings in the company at T=3. For simplicity, we assume that when a liquidity shock occurs, the manager would wish to sell all the shares she can, i.e. $\beta$ shares, even if the market were to assume that the traded shares belong to an L company and $P^3$ thus took the lowest possible value. When there is no liquidity shock, the manager will not have a liquidity reason to sell at T=3 but might decide to sell in order to make a trading profit: the manager will sell all $\beta$ shares if the manager’s estimate of the expected T=5 price is below $P^3$ and will continue to hold the shares until T=5 otherwise.

The market is assumed to know only the probability of a liquidity shock, $q$; the market is unable to observe whether a specific manager experienced a
liquidity shock. Therefore, even if the market could perfectly identify a managerial sale of shares, it still would not know whether this sale was in response to a liquidity shock or was motivated by the manager’s superior information.

Still, as is shown below, the outcome would differ depending on whether the market knows when a managerial sale of stock is taking place. For the market will be able to draw inferences from a manager’s selling of shares only if the market knows of the sale. The market’s ability to detect selling by managers would depend on two factors. First, this ability would depend on when managers are required to disclose their trading. Managers are not required to disclose in advance their intention to sell shares but only to report a sale after it was executed. Still, when managers divide their selling over a period, their disclosures over this period put the market on notice that the manager is in the process of selling. Second, even before the manager made any disclosure about past selling, actual selling might be detected by market participants to the extent that it increases the daily volume of trading. To what extent selling would be detected in this way by market makers and players would depend on the amount of shares the manager wishes to sell compared with normal trading volume.

We formally study below two polar cases. In section 4.2, we study the polar case in which the market does not detect managerial selling at all. In this case, market prices are not affected by a manager’s T=3 sale of shares, and thus \( P^3 = P^2 \). The analysis shows how managers’ ability to sell for this price at T=3 increases their incentive to invest in creating opportunities to misreport. In section 4.3, we study the opposite polar case in which the market is fully aware (at the time of sale) of any selling of shares by the manager. In this case, a manager’s T=3 selling of shares will affect market prices which would adjust to reflect the market’s inference from the fact that the manager is selling. We show that the market’s ability to draw such an inference reduces but does not eliminate the increased incentives to managers to invest in creating opportunities to misreport. Bebchuk and Bar-Gill (2002) extends the analysis to the intermediate

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19 Recent proposals by Fried (1998) to impose a pre-sale disclosure requirement are aimed to enhance the market’s ability to identify managers that are selling their shares and to draw the appropriate inferences from such sales.

20 In some companies, managers can sell their holdings to the company itself at the market price, thus sending no information to the market.

21 In the first polar case, we assume that the market is unable to identify a managerial sale of shares, regardless of the number of shares that the manager sells. In the second polar case, we assume that the market perfectly identifies all managerial sales, regardless of the number of shares sold. In a more general model the market’s ability to identify a managerial sale is
case in which managers may trade anonymously but the market can make inferences about the likelihood managers are selling from the volume of trading.

4.2 No Identification of Managerial Selling

We start with the polar case in which the market does not have (nor can infer) any information on whether the manager is actually selling shares. And we first consider the outcome that would obtain if managers were to always truthfully announce their companies’ earnings at T=2. In this case, managers would not be able to make profits from trading on superior information at T=3. Still, if a manager were to experience a liquidity shock, she would sell her holdings (to the extent permitted) at T=3.

Now consider a manager of an L company who misreported earnings at T=2. As in the truthful statements case, this manager will sell shares at T=3 when she experiences a liquidity shock. However, as we demonstrate below, if she misreports earnings at T=2, the manager will sell shares at T=3 also when she does not experience a liquidity shock. Because the manager of an L firm that misreports knows that she will be able to sell shares at T=3 for a price exceeding their expected final value, the possibility of selling shares at T=3 increases the appeal of misreporting. This is captured in the following proposition.

**Proposition 6:** In the no identification case –

(i) Managers of L companies that misreport will sell all the shares they are permitted to sell (even when they do not experience a liquidity shock).

(ii) Allowing managers to sell shares at T=3 will increase their T=1 investment in creating opportunities to misreport, and as a result the overall incidence of misreporting will increase as well.

(iii) The magnitude of the increase in investment to create opportunities to misreport, and in the overall level of misreporting, is larger when managers are permitted to sell a larger fraction of their holdings at T=3, i.e. when $\beta/\beta$ is larger.

**Remark 1(intuition):** The intuition for these results, whose detailed proof is omitted, is as follows:

(i) In the no identification case, the analysis of periods 4 and 5 remains as in section 3. Moving back in time to period 3, since managers of L companies who increasing in the number of shares sold. See, e.g., Kyle (1985), Glosten and Milgrom (1985) and the large literature that followed these seminal contributions.
can misreport elect to do so, the T=3 share price of a company that reports \( E_H \) is

\[
P^3 = \omega_0 \cdot E(P'_H) + (1 - \omega_0) \cdot E(P'_L),
\]

where \( E(P'_H) \) and \( E(P'_L) \) are the expected final period values of the shares of a type H firm and a type L firm respectively, and

\[
\omega_0 = \frac{\theta \cdot \delta}{\theta \cdot \delta + (1 - \theta)}.
\]

It can be easily shown that \( E(P'_H) > E(P'_L) \). If managers of type L companies cannot sell shares at T=3 and must wait until the final period, they will obtain an expected price of \( E(P'_L) \). If they are permitted to sell shares at T=3, they will obtain \( P^3 > E(P'_L) \). Hence, a manager of an L company who misreports at T=2 will sell at T=3 all the shares that she is permitted to sell at that stage -- even when she does not experience a liquidity shock.

As to the gain to managers from being able to sell at the intermediate period: When managers of L companies that misreport cannot sell at T=3, their expected payoff will be \( \beta \cdot E(P'_L) \). In contrast, when these managers are allowed to sell shares at T=3, their expected payoff will be \( \bar{\beta} \cdot P^3 + (\beta - \bar{\beta}) \cdot E(P'_L) \). Therefore, the freedom to sell at T=3 produces for the manager an additional gain of \( \bar{\beta} \cdot [P^3 - E(P'_L)] = \bar{\beta} \cdot (1 - \omega_0) \cdot (E(P'_H) - E(P'_L)) \) from misreporting. This extra gain induces a higher investment in creating opportunities to misreport and thus leads to a higher incidence of misreporting. The magnitude of the extra gain from misreporting, and thus also of the increase in misreporting, is increasing in \( \bar{\beta} \).

(ii) Another way of explaining the results stated in proposition 6 is as follows. When the manager of an L company that misreported at T=2 raises capital at T=4, she in fact sells a fraction of her interest in the company’s low-value initial project for the advantageous price made possible by the pooling with H companies. This “selling” is the source of the manager’s gain from misreporting in the case in which the manager is not permitted to sell shares at

\[
\text{expected final period value of type } H \text{ shares is } E(P'_H) = \gamma \cdot \Pr(R \geq \bar{R}_H) \cdot (1 - \alpha_p) \cdot V_H + K + E(R \mid R \geq \bar{R}_H) + (1 - \gamma) \cdot \Pr(R \geq \bar{R}_H) \cdot V_H, \quad \text{and the expected final period value of type } L \text{ shares of companies that reported } E_H \text{ is } E(P'_L) = \gamma \cdot (1 - \alpha_p) \cdot [V_H - \Delta V + K + \bar{R}] + (1 - \gamma) \cdot [V_H - \Delta V].
\]

Specifically, since managers of L companies who can misreport earnings at T=2 elect to do so, the expected final period value of type H shares

\[
E(P'_H) = \gamma \cdot \Pr(R \geq \bar{R}_H) \cdot (1 - \alpha_p) \cdot V_H + K + E(R \mid R \geq \bar{R}_H) + (1 - \gamma) \cdot \Pr(R \geq \bar{R}_H) \cdot V_H, \quad \text{and the expected final period value of type L shares of companies that reported } E_H \text{ is } E(P'_L) = \gamma \cdot (1 - \alpha_p) \cdot [V_H - \Delta V + K + \bar{R}] + (1 - \gamma) \cdot [V_H - \Delta V].
\]

To see this note that at period 4 managers of H companies who can misreport can always do what managers of L companies who misreport do, and earn a higher payoff than managers of L companies (since managers of H companies enjoy a higher value \( V_H > V_L \)). However, as shown in section 3, managers of H companies will adopt a different strategy, which generates an even higher payoff (otherwise, managers of H companies would do what managers of L companies do). A higher payoff for managers implies a higher final price, since at T=4 managers maximize company value.
T=3. Importantly, in this case, the manager (like the other initial shareholders) sells only a fraction of her interests in the company’s initial project for the favorable pooling price; she will still retain the same fraction of her claims to the company’s initial project as is retained by other initial shareholders. However, when the manager is permitted to sell shares at T=3, she will sell – as far as the fraction $\beta$ she is permitted to sell is concerned – all of her claims to the firm’s initial project. She will retain a smaller fraction of her initial claims to the firm’s initial project than would other initial shareholders. This would enable managers of L companies that misreported to gain more when they may sell shares at T=3.

**Remark 2 (empirical implications):** The results stated in proposition 6 are consistent with existing empirical findings, and they provide testable predictions for future empirical work:

(i) Misreporting of earnings is more likely to occur in those cases in which managers are not precluded – by law or by their compensation contract and other contracts with the firm – from selling shares in the short-run. Furthermore, misreporting is more likely to occur in sectors or companies where managers are permitted to sell a larger fraction of their initial holdings.

Bergstresser and Philippon (2002) find evidence that managers whose compensation is more directly tied to share prices are more likely to manipulate earnings (see also Yablon and Hill (2001)). Because compensation schemes generally permit managers to unload vested options (Bebchuk, Fried, and Walker (2002)), we view these findings as generally consistent with the predictions of our model.

Ke (2002) also finds that managers with stock and exercisable stock options tend to engage in earnings manipulation. He also finds no evidence that managers with large amounts of unexercisable stock tend to manage earnings. This pattern sits well with the predictions of our model: it indicates that it is not more options and shares – but rather more options and shares that the manager may sell in the short-run – that produce incentives to engage in misreporting.

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24 Put differently, when the manager of an L company cannot sell at T=3, she will still gain from misreporting – a gain that is represented by the high pooling price that a misreporting L company can get for its shares when it raises $K$ at T=4. Still the manager of the L company, if she is forced to wait until the final period, bears the smaller cash flows of an L company (lower by $\Delta V$ as compared to an H company). If the manager can sell shares at T=3, she enjoys a second pooling with the H companies. She no longer bears the entire difference in cash flows between H companies and L companies, $\Delta V$. She only bears a fraction of that difference $\omega \cdot \Delta V$. 

27
(ii) Among companies whose managers may sell the same fraction of the managers’ shares in the short run, there will be more selling by managers who subsequently are found to have misreported.

(iii) In industries with larger information asymmetries (e.g. R&D intensive industries), managers will use the freedom to sell shares earlier and will make greater profits from insider trading. This prediction is consistent with the findings of Aboody and Lev (2000).

(iv) In the voluntary disclosure interpretation of our model, there will be more selling by managers after a voluntary disclosure that turns out to be inaccurate. This prediction is consistent with the evidence (see Noe (1999)) that documents an increase in the volume of insider trading after voluntary disclosures (specifically management earnings forecasts).25

4.3 Perfect Identification of Managerial Selling

We now turn to the second polar case in which the market is fully aware of all managerial sales of shares at the time in which the sale takes place.

**Proposition 7:** In the perfect identification case, if the probability of a liquidity shock, $q$, is sufficiently high, then –

(i) Managers of L companies that can misreport will elect to do so, and at $T=3$ these managers will sell all the shares they are permitted to sell -- even when they do not experience a liquidity shock.

(ii) Compared with the case in which managers are not permitted to sell any shares at $T=3$, allowing managers to sell some shares in this period will increase their $T=1$ investment in creating opportunities to misreport, and as a result the overall level of misreporting will increase as well. The magnitude of this effect is larger when managers are permitted to sell a larger fraction of their holdings at $T=3$, i.e. when $\beta / \beta$ is larger.

(iii) The increase in the $T=1$ investment in creating opportunities to misreport, and in the overall level of misreporting, will be smaller compared with the no identification case.

---

25 Noe suggests that managers appear to be exploiting their private information (not part of the voluntary disclosure) regarding the company’s long-term prospects.
**Remark 1 (intuition):** The intuition for these results, whose proof is provided in the Appendix, is as follows:

(i) In the no-identification case, managers who misreported at T=2 will surely sell at T=3 (see proposition 6). In the perfect identification case, however, it is no longer obvious that managers who do not experience a liquidity shock will ever wish to sell at T=3. When the market cannot identify managerial selling, early T=3 sales by misreporting managers can only increase the manager’s gains. In contrast, when the market can identify managerial selling, these early sales will also impose a cost on the managers, because the sales will lead the market to revise upwards its estimate of the probability that the company is misreporting. And as long the managers are going to retain some shares, such upward revision might hurt them by worsening the terms at which the company would be able to raise equity in the event that a new project emerges.

In particular, in an equilibrium in which managers of L companies that misreport sell at T=3, the market recognizes, when facing a managerial sale, that managers of L companies that misreport always sell at T=3 while managers of H companies sell only when they experience a liquidity shock. Therefore, when observing a managerial selling of shares at T=3 by a company that announced high earnings, the market will ascribe to the possibility that the company is of type L a probability of:

\[
\hat{\omega}_0 = \frac{\theta \cdot \delta}{\theta \cdot \delta + (1 - \theta) \cdot q} \geq \omega_0.
\]

The larger the probability of a liquidity shock, q, the smaller the upward revision that the market will make in its estimate of the probability that the company is misreporting (i.e., the smaller the extent to which \(\hat{\omega}_0\) is higher than \(\omega_0\)), and the smaller the cost to the manager of the market’s inference from a managerial sale. Thus, if the probability of a liquidity shock is sufficiently high, managers who misreport will elect to sell at T=3.

(ii) When q is sufficiently large, managers of L companies who misreport at T=2 will gain from the opportunity to sell at T=3 (see remark (i)). As in the no-identification case, the magnitude of this extra gain is increasing in \(\bar{\beta}\). This added gain from misreporting will induce, to an extent that is increasing in \(\bar{\beta}\), more investment in creating opportunities to misreport and a higher incidence of misreporting.

(iii) As noted in remark (i), when the market can identify managerial selling, the inference that the market will draw from a managerial sale at T=3 will hurt the manager both in the short run and in the long run. In the short run, the manager will get a lower price at T=3 (lower \(P^3\). In the long run, the company
will have to sell more shares in order to fund a new project at T=4. Thus, the gain to managers from misreporting will be lower in the perfect identification case than in the no-identification case. It follows that the investment in creating opportunities to misreport, and the resulting level of misreporting, will be lower in the perfect identification case than in the no-identification case.

**Remark 2 (testable predictions):** The results stated in proposition 7 indicate that, other things equal, when the market can better identify managerial selling, misreporting of earnings will be less likely to occur. Specifically, the following predictions can be made:

(a) When the volume of shares offered by liquidity sellers is smaller relative to the volume of shares that managers are permitted to sell in the short run, misreporting will be less likely to occur.

(b) Tightening disclosure requirements, and in particular requiring prompt disclosure by managers following a sale of shares, will reduce the incidence of misreporting.

**Remark 3 (corporate governance implications):** Recent legislation requires managers that sell shares to disclose their sales much more quickly following the sale than was previously the case. This requirement will ensure that the market will become aware much faster of any managerial attempt to sell a substantial amount of shares over a significant period of time. Our analysis indicates that this requirement will operate to reduce the incidence of misreporting. This level of misreporting can be further reduced by an additional tightening of disclosure requirements to require in-advance disclosure of trading by managers.

However, our analysis also demonstrates that disclosure requirements, which at most can enable the market to be able to perfectly identify any managerial selling, cannot eliminate the incentives to managers to misreport and to create opportunities to misreport that arise from managers’ freedom to sell their shares. Thus, even in the presence of strong disclosure requirement, there is something that can be said for precluding or limiting managers’ ability to sell their shares in the short-run. Such limitations would discourage investments in creating opportunities to misreport and reduce the incidence of misreporting.

5. Related Literature

Our analysis of the causes and consequences of misreporting is related to, and builds on, several lines of work in the economics, finance, and accounting
literatures. To begin, there is a large body of literature -- both theoretical and empirical -- on disclosure by firms. Verrecchia (2001), Dye (2001), Healy and Palepu (2001), Core (2001), Fishman and Hagerty (1998) and Gertner (1998) offer good surveys of this literature. The disclosure literature, however, has largely assumed that reports made by companies are truthful (see Verrecchia (2001), pp. 142-143). This assumption is already used in early, classic works by Grossman and Hart (1980), Grossman (1981), Milgrom (1981) and Jovanovic (1982), and continues to be used by most recent contributions (see, e.g., Admati and Pfleiderer (2000)).

Although the disclosure literature focuses on truthful disclosure, several models developed by it have allowed for false reporting by firms (see, e.g., Dye (1988), Benabou and Laroque (1992), Arya et al. (1998), Verrecchia (2001), and Lambert (2001, sec. 5). However, these models differ from ours in some of the key elements that it includes, such as the endogenous determination of opportunities to misreport, the explicit link between misreporting and financing and investment decisions, and the explicit link between managers’ compensation contracts and holdings and their disclosure decisions.

Another important and related line of work focuses on the adverse effects that asymmetric information has on investment and financing decisions. In their classic work, Myers and Majluf (1984) show how the fear of pooling with low value companies might prevent high value companies from selling equity in order to finance new projects. The subsequent literature has studied in detail the effects of various information structures on the decisions whether and when to have an equity offering (see, e.g., Korajczyk, Lucas & McDonald (1991, 1992) and Lucas & McDonald (1990, 1992)). This literature has also considered the effects of asymmetric information on the issuing of equity in the context of a stock acquisition.

Among other things, researchers have looked both at how the decision to use cash or equity to pay for an acquisition is influenced by over-valuation or under-valuation of the acquirer’s stock by the market and at the inferences that investors might draw from such decisions (see, e.g., Shleifer and Vishny (2001), Jovanovic and Braguisnksy (2002)). We incorporate the effects of asymmetric information and capital and investment decisions into our analysis and connect them to the problem of misreporting. In our model, the level of asymmetric information, which affects decisions whether to raise equity to finance an acquisition of a new project, is determined endogenously as a function of companies’ investments in

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26 Admati and Pfleiderer (2000) allow for inaccurate disclosure, but not for false disclosure. See also Fishman and Hagerty (1990).
creating opportunities to misreport and their decisions whether to use such opportunities.

Yet another line of related work is that on managerial short-termism and myopia (see, e.g., Stein (1988, 1989) and Bebchuk and Stole (1993)). In this literature, managers’ choice between long-term and short-term projects is affected by their concern about short-run prices. In our model, the focus is not on the choice between long-term and short-term projects, but rather on investments in creating opportunities to misreport and on decisions whether to use such opportunities. Furthermore, whereas in the myopia literature managers are assumed to give some exogenously stipulated weight to short-run and long-run stock prices, our model includes an explicit derivation of why managers might give some weight to short-run prices. In particular, we show how (and how much) concern about short-run stock prices might arise from managers’ desire to raise equity for the firm at favorable terms or from their desire to sell some of their shares on the market.

Finally, in terms of corporate governance, our analysis confirms concerns that have been expressed informally about the possible links between agency problems and problems of misreporting (see, e.g., Healy and Palepu (1993) and references cited therein). In particular, our analysis (like that of Benabou and Laroque (1992)) confirms concerns that short-term components of executive compensation (including provisions allowing for early selling of company stock by managers) can have an adverse effect on problems of misreporting.

6. Conclusion

We have developed in this paper a model of the causes and consequences of corporate misreporting. The model is sufficiently rich to include many elements of interest, including the link between misreporting and capital and investment decisions, and the connection between misreporting, managers’ compensation, and insider trading. Our analysis can help explain the rise in misreporting in the late 90’s. It also provides a wide range of testable predictions, some of which are consistent with existing evidence and some of which can provide a basis for future empirical work. Our analysis also highlights the real economic costs of misreporting, and it identifies which corporate governance and compensation arrangements can make misreporting more or less likely. Requiring managers to make pre-trading disclosure of their plans to sell shares, or even restricting them from selling shares in the short-run, would decrease – but eliminate -- the incidence and costs of misreporting.
Appendix

The appendix collects the proofs of many of the propositions and corollaries presented in the text.

Proof of Proposition 1:
(i) See remark following proposition 1.
(ii) The threshold value \( \hat{\theta} \) satisfies:

\[
(1) \quad (1 - \alpha) \left[ V_H + K + \hat{\theta} \right] = V_H,
\]

where \( \alpha \) is the fraction of the company that needs to be sold to raise \( K \). Note that (1) implies:

\[
\alpha = \frac{K + \hat{\theta}}{V_H + K + \hat{\theta}} \quad \text{(and } \frac{1}{\alpha} = 1 + \frac{V_H}{K + \hat{\theta}} \text{)}.
\]

The fraction, \( \alpha \), also satisfies:

\[
(2) \quad \alpha \cdot \Pi = K.
\]

where \( \Pi \) is the average value of a company that reports \( E_H \) and sells equity (namely, given that a company reported \( E_H \) and is offering equity, the market knows that the average value of the company is \( \Pi \)).

This average value is given by:

\[
(3) \quad \Pi = V_H - \left( \theta \cdot \delta \cdot \rho \right) \cdot \Delta V + K + \left[ \theta \cdot \delta \cdot R + (1 - \theta) \cdot \Pr \left( R \geq \hat{\theta} \right) \cdot E \left( R \mid R \geq \hat{\theta} \right) \right] / \rho,
\]

where \( \rho = \theta \cdot \delta + (1 - \theta) \cdot \Pr \left( R \geq \hat{\theta} \right) \) is the probability that a company sells equity (given that a new project is available). The average value of a company that sells equity can also be written as:

\[
\Pi = V_H + K + \frac{\int_{\hat{\theta}}^{\infty} R \cdot f(R) \cdot dR - \theta \cdot \delta \cdot \Delta V}{\theta \cdot \delta + (1 - \theta) \cdot \left( 1 - F \left( \hat{\theta} \right) \right)}.
\]

Combining (1) and (2), we obtain:

\[
V_H + \frac{\int_{\hat{\theta}}^{\infty} R \cdot f(R) \cdot dR - \theta \cdot \delta \cdot \Delta V}{\theta \cdot \delta + (1 - \theta) \cdot \left( 1 - F \left( \hat{\theta} \right) \right)} = \frac{K \cdot V_H}{K + \hat{\theta}},
\]

or

\[
\theta \cdot \delta \cdot \bar{R} + (1 - \theta) \cdot \int_{\hat{\theta}}^{\infty} R \cdot f(R) \cdot dR - \theta \cdot \delta \cdot \Delta V
\]

\[
= \frac{V_H \cdot \hat{\theta}}{K + \hat{\theta}}.
\]

QED
Proof of Corollary 1:

(i) From equation (1) we know that an H company facing a new project with R project will be willing to sell (up to) a fraction \( \alpha(R) = \frac{K + R}{V_h + K + R} \) of its shares to finance the new project. From equation (2) we know that this company will need to sell a fraction \( \alpha(\hat{R}_h) = \frac{K}{\Pi(\hat{R}_h)} \) of its shares to finance the new project.

When \( \omega_0 \cdot \Delta V \leq \bar{R} \), \( \alpha(\hat{R}_h = 0) \leq \bar{\alpha}(R = 0) \) (from equation (3) we know that \( \Pi(\hat{R}_h = 0) = V_h + K - (\omega_0 \cdot \Delta V - \bar{R}) \geq V_h + K \), where \( \omega_0 = \frac{\theta \cdot \delta}{\theta \cdot \delta + (1 - \theta)} \). Hence, even when facing a project with \( R = \hat{R}_h = 0 \) an H company will finance and invest in the new project.\(^{27}\)

When \( \omega_0 \cdot \Delta V > \bar{R} \), \( \alpha(\hat{R}_h > 0) > \bar{\alpha}(R = 0) \). Hence, we cannot have \( \hat{R}_h = 0 \). To prove that there exists a threshold \( \hat{R}_h > 0 \), we first show that if \( \hat{R}_h \) is sufficiently large, then \( \alpha(\hat{R}_h) \) is decreasing in \( \hat{R}_h \). This ensures that there exists a threshold \( \hat{R}_h > 0 \), such that \( \alpha(\hat{R}_h) \leq \bar{\alpha}(R \geq \hat{R}_h) \). Rearranging equation (3), we obtain:

\[
\Pi(\hat{R}_h) = V_h + K + \theta \cdot \delta \cdot \bar{R} + (1 - \theta) \cdot \Pr(R \geq \hat{R}_h) \cdot E[R|R \geq \hat{R}_h] - \theta \cdot \delta \cdot \Delta V \Big/ \rho .
\]

For a sufficiently large \( \hat{R}_h \), the bracketed term is positive and increasing in \( \hat{R}_h \). Thus, since \( \rho \) is decreasing in \( \hat{R}_h \), \( \Pi(\hat{R}_h) \) is increasing in \( \hat{R}_h \), and \( \alpha(\hat{R}_h) \) is decreasing in \( \hat{R}_h \).

(ii) Given \( \omega_0 \cdot \Delta V > \bar{R} \) and a positive \( \hat{R}_h \), we can use equation (4) to obtain the partial derivative of \( \hat{R}_h \) with respect to \( \Delta V \):

\[
\frac{\partial \hat{R}_h}{\partial \Delta V} = \frac{\rho \cdot \theta \cdot \delta}{\Psi} > 0 ,
\]

where

\[
\Psi = \rho^2 \cdot K \cdot V_h / \left( (K + \hat{R}_h)^2 - \rho \cdot (1 - \theta) \cdot f(\hat{R}_h) \cdot \hat{R}_h \cdot (1 + V_h / (K + \hat{R}_h)) \right) .
\]

We assume that \( \Psi > 0 \). The condition \( \Psi > 0 \) will hold as long as the possible values of \( R \) are more or less evenly distributed over \([0, \infty)\), so that \( f(R) \) is never too large.

The numerator of the derivate \( \frac{\partial \hat{R}_h}{\partial \Delta V} \) represents the direct effect of \( \Delta V \) on the average expected value, \( \Pi \). The first element in \( \Psi \), \( \rho^2 \cdot K \cdot V_h / \left( (K + \hat{R}_h)^2 \right) \),

\(^{27}\) If an H company facing a new project with \( R = \hat{R}_h = 0 \) decides to invest, then clearly an H company facing a new project with \( R > \hat{R}_h = 0 \) will decide to invest. To see this note that \( \alpha(\hat{R}_h) \) is independent of \( R \), and \( \bar{\alpha}(R) \) is increasing in \( R \).
represents the reluctance of H companies to sell equity, namely the higher $\hat{R}_H$, that follows from the need to sell more shares (in order to raise $K$) due to the cross subsidization effect. The second element in $\Psi$, $-\rho \cdot (1-\theta) \cdot f(\hat{R}_H) \cdot \hat{R}_H \cdot \left(1 + V_H / (K + \hat{R}_H)\right)$, represents the second order effects of raising $\hat{R}_H$ on $\Pi$ and $\rho$.

(iii) See remark (iii) following corollary 1.

QED

Proof of Proposition 3:
We formally prove part (iii) of proposition 3. The proof of the remaining parts of the proposition is immediate from the remarks following proposition 3.

To show that $C^*$ is increasing in $\Delta V$, we need to show that the gain from misreporting $G = \gamma \cdot (\alpha_L - \alpha_p) \cdot (V_L + K + \bar{R})$ is increasing in $\Delta V$, namely we need to show that $\frac{\partial \alpha_p}{\partial (\Delta V)} < 0$. From equations (2) and (3), we obtain: $\alpha_p = \frac{K}{\Pi}$, where

$$\Pi = V_L + (1-\theta \cdot \delta / \rho) \cdot \Delta V + K + \left[ \theta \cdot \delta \cdot \bar{R} + (1-\theta) \cdot \int_{\hat{R}_H}^{\infty} R \cdot f(R) \cdot dR \right] / \rho$$

and

$$\rho = \theta \cdot \delta + (1-\theta) \cdot \left(1 - F(\hat{R}_H)\right) - \left(1 - F(\hat{R}_H)\right)$$.

Therefore, we need to show that $\frac{\partial \Pi}{\partial (\Delta V)} > 0$. The direct effect of $\Delta V$ on $\Pi$ (i.e. $(1-\theta \cdot \delta / \rho)$) is positive. The indirect effect of $\Delta V$ on $\Pi$, operating through the effect of $\Delta V$ on $\hat{R}_H$, can be either positive or negative. But, as long as $f(\hat{R}_H)$ is not too large, the direct effect is dominant, and thus $\frac{\partial \Pi}{\partial (\Delta V)} > 0$.

QED

Proof of Proposition 5:
(i) L companies that cannot misreport will clearly forgo any inefficient project. The question is whether L companies that can misreport will sell equity when facing an inefficient project. L companies that can misreport will elect to do so (see proposition 1). To ascertain whether these L companies, when facing an inefficient project, will sell equity, we first derive the profits of such L companies in the following two cases – (a) L companies that can misreport always sell equity (even when facing an inefficient project), and (b) L companies that can misreport sell equity only when facing an efficient project.
(a) L companies always sell equity: L companies facing an inefficient project will sell equity and enjoy a payoff of $(1-\alpha_p)\left(V_L + K - \bar{R}\right)$. To find the threshold value $\hat{R}_H$, we follow the procedure used in the basic model. Equations (1), (2) and (3) still hold, subject to the following new definition:

$$\bar{R} = (1-p)\cdot\int_0^\infty R \cdot f(R) \cdot dR - p \cdot \bar{R}.$$ 

Hence, equation (4) still holds as well (with the new definition).

(b) L companies sell equity only when facing an efficient project: L companies facing an inefficient project will not sell equity, thus earning a payoff of $V_L$.

Comparing the payoff in case (a) and the payoff in case (b), we find that L companies that can misreport will sell equity when facing an inefficient project if and only if $(1-\alpha_p)\left(V_L + K - \bar{R}\right) > V_L$, or $\alpha_p < \frac{K - \bar{R}}{V_L + K - \bar{R}}$. From equation (2), we know that $\alpha_p = \frac{K}{\Pi}$. Therefore, L companies that can misreport will sell equity when facing an inefficient project if and only if

$$(5) \quad \frac{K}{\Pi} < \frac{K - \bar{R}}{V_L + K - \bar{R}}.$$ 

Since $\Pi = V_L + (1-\theta)\cdot\Delta V + K + \left[\theta \cdot \delta \cdot \bar{R} + (1-\theta) \cdot \text{Pr}(R \geq \hat{R}_H) \cdot E[R | R \geq \hat{R}_H] \right] / \rho$, when $\Delta V$ is sufficiently high, $\Pi$ will become sufficiently high so that condition (5) is satisfied.

(ii) We first show that $\hat{R}_H$ is larger when L companies undertake inefficient projects, or more generally when $p$ is larger. With inefficient projects, equation (4) becomes:

$$\frac{\theta \cdot \delta \cdot \left(1-p\right) \cdot \int_0^\infty R \cdot f(R) \cdot dR - p \cdot \bar{R} + (1-\theta) \cdot \int_{\hat{R}_H}^\infty R \cdot f(R) \cdot dR - \theta \cdot \delta \cdot \Delta V}{\theta \cdot \delta + (1-\theta) \cdot (1-F(\hat{R}_H))} + \frac{V_H \cdot \hat{R}_H}{K + \hat{R}_H} = 0.$$ 

We take the derivative of equation (4) (which defines $\hat{R}_H$) with respect to $p$:

---

28 The fraction $\alpha_p$ is defined as before, subject to the appropriate adjustments.

29 As noted in the proof of proposition 3, the direct effect of $\Delta V$ on $\Pi$ (i.e. $(1-\theta \cdot \delta / \rho)$) is positive. The indirect effect of $\Delta V$ on $\Pi$, operating through the effect of $\Delta V$ on $\hat{R}_H$, can be either positive or negative. But, as long as $f(\hat{R}_H)$ is not too large, the direct effect is dominant.
\[ \frac{\partial \hat{R}_H}{\partial \hat{p}} = \frac{\rho \cdot \theta \cdot \delta \cdot \left( \int_{0}^{\infty} R \cdot f(R) \cdot dR + \hat{R} \right)}{\Psi} > 0. \]

Next, we take the derivative of equation (4) (which defines \( \hat{R}_H \)) with respect to \( \hat{R} \):

\[ \frac{\partial \hat{R}_H}{\partial \hat{p}} = \frac{\rho \cdot \theta \cdot \delta \cdot \hat{p}}{\Psi} > 0. \]

QED

**Proof of Proposition 7:**

(i) As in the no identification case, if a manager of an L company truthfully announces her company’s earnings at T=2, she will sell shares at T=3 only when she experiences a liquidity shock. However, contrary to the no identification case, a manager of an L company, who can misreport earnings at T=2 and elects to do so, will not necessarily sell shares at T=3.

Assume that managers of L companies, who can misreport, elect to do so and sell at T=3. Since managers of H companies sell at T=3 only when they experience a liquidity shock, the market learns from the T=3 sales. In particular, the market infers the probability that the company is of type L. When observing a managerial selling of shares at T=3 (by a manager of a company that reported \( E_{Ht} \)), the market knows that the probability that the company is of type L is:

\[ \hat{\omega}_0 = \frac{\theta \cdot \delta}{\theta \cdot \delta + (1-\theta) \cdot q} \geq \omega_0. \]

Consequently, the T=3 selling lowers the final period value of both type H shares and type L shares. Specifically, when managers of L companies who can misreport earnings at T=2 elect to do so, the expected final period value of type H shares, \( E(P_{Lt}') \), becomes:

\[ E(P_{Lt}'(\hat{\omega}_0)) = \gamma \cdot \Pr(R \geq \hat{R}_H(\hat{\omega}_0)) \cdot (1-\alpha_p(\hat{\omega}_0)) \cdot V_H + K + E[\bar{R}|R \geq \hat{R}_H(\hat{\omega}_0)] + (1-\gamma) \cdot \Pr(R \geq \hat{R}_H(\hat{\omega}_0)) \cdot V_H \]

where \( \alpha_p(\hat{\omega}_0) \geq \alpha_p(\omega_0) \) and \( \hat{R}_H(\hat{\omega}_0) \geq \hat{R}_H(\omega_0) \). And, the expected final period value of type L shares, \( E(P_{Lt}') \), becomes:

\[ E(P_{Lt}'(\hat{\omega}_0)) = \gamma \cdot (1-\alpha_p(\hat{\omega}_0)) \cdot [V_H - \Delta V + K + \bar{R}] + (1-\gamma) \cdot [V_H - \Delta V]. \]

The T=3 price is: \( P^3(\hat{\omega}_0) = \hat{\omega}_0 \cdot E(P_{Lt}'(\hat{\omega}_0)) + (1-\hat{\omega}_0) \cdot E(P_{Lt}'(\hat{\omega}_0)) \). If managers of L companies who can misreport earnings at T=2 elect to do so, and sell \( \bar{\beta} \) at T=3, their expected payoff is:

\[ \bar{\beta} \cdot P^3(\hat{\omega}_0) + (\beta - \bar{\beta}) \cdot E(P_{Lt}'(\hat{\omega}_0)). \]

On the other hand, if managers of L companies who can misreport earnings at T=2 elect to do so, but do not sell shares at T=3, their expected payoff is:
Comparing the two expected payoffs, the manager of an L company, who misreported earnings at T=2, will always sell at T=3 if and only if
\[ \bar{\beta} \cdot P^3(\hat{\omega}_0) + (\beta - \bar{\beta}) \cdot E(P'_L(\hat{\omega}_0)) \geq \beta \cdot E(P'_L(\omega_0)), \]
or
\[ (6) \quad \frac{\bar{\beta}}{\beta} \geq \frac{E(P'_L(\omega_0)) - E(P'_L(\hat{\omega}_0))}{P^3(\hat{\omega}_0) - E(P'_L(\hat{\omega}_0))}. \]
This condition will always hold when \( q \) is sufficiently large (note that \( \lim_{q \to 1}(E(P'_L(\omega_0)) - E(P'_L(\hat{\omega}_0))) = 0 \)).

(ii) When managers are allowed to sell shares at T=3, they enjoy an extra gain of
\[ \bar{\beta} \cdot P^3(\hat{\omega}_0) + (\beta - \bar{\beta}) \cdot E(P'_L(\hat{\omega}_0)) - \beta \cdot E(P'_L(\omega_0)), \]
or
\[ \bar{\beta} \cdot (P^3(\hat{\omega}_0) - E(P'_L(\hat{\omega}_0))) - \beta \cdot (E(P'_L(\omega_0)) - E(P'_L(\hat{\omega}_0))) \]
from misreporting earnings. When condition (6) holds, this gain is positive. Moreover, this gain is increasing in \( \bar{\beta}/\beta \).

(iii) Comparing the extra gain derived in part (ii) above to the extra gain calculated in the remarks following proposition 6, we know that the former is smaller if and only if
\[ \bar{\beta} \cdot (P^3(\hat{\omega}_0) - E(P'_L(\hat{\omega}_0))) - \beta \cdot (E(P'_L(\omega_0)) - E(P'_L(\hat{\omega}_0))) < \bar{\beta} \cdot (P^3(\omega_0) - E(P'_L(\omega_0))), \]
or
\[ \bar{\beta} \cdot (P^3(\omega_0) - P^3(\hat{\omega}_0)) - \bar{\beta} \cdot E(P'_L(\omega_0)) + \beta \cdot E(P'_L(\omega_0)) - \beta \cdot E(P'_L(\hat{\omega}_0)) + \bar{\beta} \cdot E(P'_L(\hat{\omega}_0)) > 0, \]
or
\[ \bar{\beta} \cdot (P^3(\omega_0) - P^3(\hat{\omega}_0)) + (\beta - \bar{\beta}) \cdot E(P'_L(\omega_0)) - E(P'_L(\hat{\omega}_0)) > 0. \]
This condition is always satisfied, since \( P^3(\hat{\omega}_0) \leq P^3(\omega_0) \) and \( E(P'_L(\hat{\omega}_0)) \leq E(P'_L(\omega_0)) \).

QED

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30 If this condition does not hold, then we have a mixed-strategy equilibrium, where some managers of L companies (who misreported earnings at T=2) always sell at T=3, and some sell only when they experience a liquidity shock. Note that in such an equilibrium, the probability of a liquidity shock, \( q \), has a direct effect on the incentive to misreport earnings – a higher \( q \) increases the probability that the manager will enjoy the higher T=3 price.
References


