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Modeling Collegial Courts (3): Adjudication Equilibria

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Abstract

We present a formal game theoretic model of adjudication by a collegial court. The model incorporates dispute resolution as well as judicial policy making and indicates the relationship between the two. It explicitly addresses joins, concurrences and dissents, and assumes “judicial” rather than legislative or electoral objectives by the actors. The model makes clear predictions about the plurality opinion’s location in “policy” space; the case’s disposition; and the size and composition of the disposition-, join-, and concurrence-coalitions. These elements of adjudication equilibrium vary with the identity of the opinion writer and with the location of the case. In general, the opinion is not located at the ideal policy of the median judge. The model suggests new directions for empirical work on judicial politics.

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This paper substantially revises a prior post "Modeling Collegial Courts (3): Judicial Objectives, Opinion Content, Voting and Adjudication Equilibria." The current version presents the model more clearly but lacks a number of illustrative examples and figures that appear in the prior version.
1. INTRODUCTION

Twenty years ago, positive political theorists began to adapt models developed for the study of legislatures and elections to the study of courts and adjudication. These models, though they have provided great insight into adjudication, largely transfer to courts the assumptions about agenda setting, voting protocols, and objectives used in the study of legislatures. Courts, however, are not legislatures and judges are not legislators. We believe that further progress requires more attention to the institutional features that actually distinguish courts – especially collegial courts – from legislatures.

In this essay, we focus on three distinctive features of adjudication on collegial courts. We offer a simple model of these institutional structures and contrast it with the structure of typical models of legislation and elections. First, any court, whether collegial or not, jointly announces a disposition of the case – whether plaintiff prevails or not – and a policy or legal rule. The announced legal rule, when applied to the facts of the case, must dictate the actual disposition of the case. The joint production of dispositions and rules requires a model of adjudication grounded in a case space.

Second, the majority disposition of a case need not attract a majority opinion. Though a majority of judges may agree that plaintiff should prevail, this majority may disagree about the rule

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1 We thank seminar participants at NYU’s Political Economy Workshop; Columbia University’s Political Economy Seminar; the University of Southern California Law School; the 3rd Annual Triangle Law and Economics Conference, Duke University; the 2009 meeting of the Association of Public Economics Theory; the 2009 Comparative Law and Economics Forum meeting; the 4th Annual Asian Law and Economics Association meeting; and the 2008 American Political Science Association meeting.
that should govern this class of cases. Phrased differently, courts – at least US courts – do not always have majority opinions, and even when they do, they also may have concurrences and dissents.²

Third, the objectives of judges who write opinions differ from the objectives usually attributed to contending candidates in electoral politics. Judges do not aim at winning \textit{per se}; rather, they care about the disposition of the case and the rule announced by the court. Obviously, the most appropriate specification of the judicial objective function will be controversial. We take a first cut at particularly "judicial" objectives by assuming that the justice who writes an opinion cares about the policy expressed in the opinion, the extent of support her opinion attracts, and the resolution of the dispute before the Court.

The introduction of these three features of adjudication³ points to the importance of several phenomena not generally addressed in the prior literature. In particular, the model presented below makes predictions about the case disposition, the content of the opinion, and the structure of the "winning" coalition including joins and concurrences. Dispositions, case content, and the structure of the winning coalition vary with the ideal policy of the opinion writer and with the location and importance of the case.

\footnotesize

² Courts in other countries have different practices that require different strategies of modeling. In France, for example, opinions of the \textit{Cour de cassation} are unanimous (and unsigned). In the British House of Lords, by contrast, each judge announces her own opinion.

³ We ignore a fourth important difference between legislation and adjudication: courts have limited control over their agenda. Litigants decide whether to bring and prosecute a case and any appeal. Judicial rulings on policy are thus constrained to policies connected to the cases before them. Legislators do not face any corresponding constraint.
It is illuminating to compare judicial procedures with electoral and legislative ones. Both electoral and legislative procedures differ from judicial ones in the respect noted above: courts jointly dispose of a case and craft a legal rule. Elections select winning candidates from a slate while legislatures enact statutes pursuant to complex procedures; in neither case is there simultaneous production of a policy and resolution of a dispute. The comparison is most revealing when we analogize judicial rules to electoral candidates and to legislative bills. We return to the phenomenon of joint production at the end of this subsection.

The identification of opinions with candidates implies content competition among opinions. From this perspective, (to use Myerson’s terminology [1999]), the “election” over the candidate opinions involves single-positive voting in “joins,” with a majority quota requirement for achieving an “opinion of the Court.” However, in the event no opinion obtains an absolute majority, there is no runoff between the leading plurality opinions. A plurality opinion on the losing side of the case disposition is never a potential winner, but the opinion with the largest plurality (but not a majority) on the winning side of the disposition is not clearly the “winning” opinion either; the content of the law has changed but the precise formulation of the legal rule that prevails is controversial.  

Alternatively, the opinions may be analogized to bills considered by a legislature. In this view, the opinions’ rules are equivalent to the bills’ policy content. From this perspective, multiple “bills” are presented to the “floor” (the Court) under a closed rule. In contravention to standard Anglo-American legislative procedure, all submitted “bills” are voted on simultaneously; the court does not hear motions, hold formal debates, or produce formal votes on motions to amend. A “join” is similar to a “yea” on a final passage vote, however declining to join is closer to an abstention than a “nay.” If an opinion/bill achieves an absolute majority in joins from the justices participating, it is

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4 Kornhauser and Sager 1993 (pp. 45-48) reviews several positions and cites relevant case law.
“enacted”. If no opinion achieves a majority in joins, the outcome is again complex. A plurality winner on the losing side of the disposition cannot be considered as “enacted.” If a single opinion accrues a plurality of votes from the disposition majority it probably should be considered “enacted.” If multiple opinions on the winning side of the disposition achieve a plurality of joins, one might argue all are enacted.

It is worth noting the role of the “status quo” or “reversion” policy in light of these procedures. For many cases considered by high courts, no definite legal rule has been announced previously. So no status quo rule may exist at all. But even if there is a standing rule applicable to the instant case, each justice can consider only the rules contained in the opinions before her. If the pre-existing rule is not offered in an opinion, the members of Court have no mechanism for endorsing that rule. Moreover, one or more of the opinions supporting the majority disposition inevitably achieves at least a plurality of the majority side joins. The doctrinal guidance that ensues may be crystal clear or completely muddy, but it is what it is: once the Court takes up a case, doctrine never reverts to the status quo ante.

In light of these complex and somewhat confusing procedures, some analysts focus on the case disposition as the “outcome” of adjudication, since dispositions are clear. This approach then conflates majority-side joins and concurrences as equivalent votes for the majority disposition, and minority-side joins and dissents as equivalent votes for the other disposition. Although clear, this approach ignores legal doctrine. By ignoring the policy-making component of adjudication, this approach does considerable violence to the intent and consequences of judicial decision making, particularly in appellate courts.

A subject of long-standing controversy is the judicial utility function (Baum 1998, Posner 1993). In light of the above discussion, three elements seem plausible as arguments: 1) preferences about rules (policy making), 2) preferences about case dispositions and, arguably, 3) preferences for clarity in the law (so that, ceteris paribus, a larger majority joining a rule is better than a smaller
majority). To the extent the first motivation weighs heavily, justices are single-minded seekers of legal policy; to the extent the second is pre-eminent, they are piece-meal dispensers of justice; and to the extent the third holds sway, they are maximizers of legal clarity.

A further complication, however, involves the extent to which judges are outcome-oriented versus act-oriented. For example, should a judge endorse a relatively poor rule rather than a better one, because a vote for the latter will just be “wasted” while a vote for the former will create a plurality winner? In crafting a rule for the consideration of her colleagues, should a judge deviate dramatically from what she sees as the best rule simply to garner votes? Should she offer a rule yielding the wrong disposition in the instant case, if that is the only way to secure a winning opinion? As sophisticated players of adjudication games, judges are certainly capable of outcome-oriented calculations; but it is not at all clear that judges (in contrast with voters, candidates, and legislators) view such calculations as proper or appropriate.

The discussion proceeds as follows. Section 2 briefly reviews the current state-of-the-art in modeling adjudication games. Section 3 presents the model. Section 4 details equilibria. Section 5 investigates the behavior of the model utilizing monotone comparative statics and the calculation of equilibria in benchmark examples. Section 6 concludes. An appendix contains several proofs. Many of the issues raised in this paper are novel, so we strive for simplicity throughout.

2 ADJUDICATION GAMES IN THE EXISTING LITERATURE

As noted in the introduction, the positive political theory of adjudication generally conflates the vote on the disposition of the case and the decision to join an opinion. This confusion is clearest in the empirical methodology that estimates the ideal policy points of justices from data on the justices’ votes on dispositions and then uses the estimated ideal points to discuss preferences about
policy. In the theoretical literature, the parallel phenomenon simply ignores the disposition of the case as the judges are presumed to have preferences over policies and simply to choose policies.⁵

With the exception of two papers discussed below, the literature ignores case disposition and focuses on the location of judicial policy. Indeed, the empirical and theoretical literature identifies, often informally, four possibilities for opinion location: 1) at the ideal policy of the median justice (the “median justice model”), 2) at the ideal policy of the median justice in the dispositional majority (the “majority median model”, 3) at the most-preferred policy position of the author of the opinion (the “author monopoly model”); and 4) at a policy position somewhere between the ideal policy of the median justice and the ideal policy of the author (“author influence models”). We compare our results with these heuristic benchmarks.

Even if one accepts the restriction of analysis to the announcement of policy, however, the literature still ignores the institutional peculiarities of policy announcement by courts. Hammond et al (2005) consider several models of Supreme Court adjudication but each simply transfers some aspect of legislative practice to the judicial context. An “open-bidding” model of bargaining and a “median hold-out” model of bargaining are consonant with standard models of legislative bargaining; each implies that policies are announced at the ideal policy of the median justice. A third model adapts models of legislative agenda-setting to the judiciary. In legislative models, the agenda-setter is sometimes a monopolist as Hammond et al assume. In this model, the agenda setter influences the location of judicial policy.

⁵ A small but growing literature begins in “case” space and then defines policy space in terms of this underlying case space. The use of case space permits a distinction between announcing policy and disposing of the case. Some models remain purely in case space – see e.g. Cameron and Kornhauser [2006, 2007] – and hence deal solely with case dispositions. Much of this literature, however, focuses solely on the announcement of policy and largely ignores case disposition.
Lax and Cameron (2007) construct an author influence model. In their model, the Court operates as a legislature under an open rule but there is a cost to writing an opinion. Moreover, the clarity and precision of the opinion depends on the effort invested in writing it. This cost raises a barrier to entry that allows the opinion writer to move the opinion away from the median and towards her own ideal point. Case dispositions are implied by opinion locations and case location, but play no real role in the analysis.

Fischman (2008) models the votes on disposition and ignores policy. In some respects, this model is closest in spirit to the one offered here. In his model as in ours, there is a one-dimensional case space and a corresponding one-dimensional policy space defined by a cut-point. The location of the case relative to the judge’s cut point determines her sincere view of the correct disposition of the case. As in our model, endorsing a disposition different from one’s sincere view of the disposition imposes a cost on the judge. In our model this loss is a constant, in Fischman’s model it is linear in the distance between the case location and the judge’s ideal point. Dissent imposes a cost both on the dissenting judge (as it does in our model) and on the two majority judges. This latter cost does not appear in our model. In Fischman’s model as in ours, a judge may vote strategically on the disposition of the case.

Carrubba et al (2008) offer a majority median model in the first model to encompass both a vote on the disposition and a vote on policy. The model there is in some respects more ambitious and in other respects less ambitious than ours. Their model, more fully than ours, acknowledges the importance of opinions that are joined by at least a majority of judges; the majority opinion is thus a public good, a fact which presents difficult analytic problems involving pivot calculations and free riding. On the other hand, to focus on the median of the majority outcome, the model makes two less ambitious assumptions. First, it restricts attention to situations in which each justice sufficiently values dispositions to insure that each judge votes for the disposition she most favors. Second, the model transplants essentially legislative institutions for voting to the judicial context; it
is these assumptions that lead to the median of the majority outcomes that do not arise in our model.

3 THE MODEL

3.1 CASES, RULES, DISPOSITIONS, AND OPINIONS
Important building blocks of the model are cases, rules, dispositions, and opinions, concepts which we now formalize.

The fact or case space is the unit interval \( \hat{X} = [0,1] \). A case \( \hat{x} \) is a distinguished element of the case space \( \hat{X} \). The content of an opinion is a “rule,” a function that maps cases into dispositions: given the facts in the case, a rule produces a “correct” disposition. Dispositions are dichotomous, i.e. “for Plaintiff” or “for Defendant.” In our simplified model, we assume rules take the following form

\[
r(\hat{x}, x) = \begin{cases} 
0 & \text{if } \hat{x} < x \\
1 & \text{if } \hat{x} \geq x 
\end{cases}
\]  

(1)

where 0 indicates one disposition and 1 indicates the other. In words, a rule employs a cut-point \( x \) establishing two equivalence classes in the case space with respect to dispositions. For instance, a rule may establish a minimal standard of care, a maximum level of acceptable intrusiveness in a government search, a speed limit, a maximum level of entanglement of state operations with religion, and so on. Using the rule, all cases in which (for instance) the actual level of care \( \hat{x} \) is less than the standard \( x \) are to receive one disposition, while all cases in which the actual level of care meets or exceeds the standard are to receive the other. Although we simplify considerably, legal rules often take this form (see, e.g., Twining and Miers 1999).
Given this simple structure for rules, each rule can be indexed by its cut-point; in this special case, policy space is isomorphic to case space.\(^6\) And, the content of each opinion corresponds to the cut-point of the rule it proposes. Accordingly, we denote rule content by \(x \in X\). (For clarity of exposition we distinguish the case space \(\hat{X}\) from the opinion space \(X\) though here both are the unit interval.)

### 3.2 PLAYERS, ACTIONS, SEQUENCE OF PLAY, STRATEGIES, AND OUTCOMES

The players are the nine justices, one of whom acts as opinion writer. The remaining non-writing justices make join decisions and cast votes on the case disposition. When referring to a justice as writer we employ subscript \(j\); when referring to any other justice we employ subscript \(i\).

The opinion writer determines the content of the opinion, \(x_j \in X\), the spatial location of his candidate rule’s cut-point. As explained previously, the opinion location in tandem with the case location \(\hat{x}\) implies a case disposition associated with the opinion, \(r(\hat{x}, x_j)\). Each justice must vote on the case disposition and may or may not join the opinion, effectively endorsing its content. A non-writing justice’s action is thus defined by two components, 1) a dispositional vote \(d_i \in D = \{0, 1\}\) (e.g. “for Defendant”, “for Plaintiff”), and 2) a join decision \(s_i \in S = \{0, 1\}\) (i.e., not join,

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\(^6\) More generally, policy space is a set of allowable partitions of case space. Not all understandings of allowable partitions yields an isomorphism between case space and policy space. Consider a set of policies governing allowable speeds on limited access highways. Case space consists of the speed at which the individual drives; we may normalize this to the interval \([0, 1]\). We might consider policies characterized by two numbers: a minimum speed and a maximum speed. Policy space then consists of all partitions of \([0, 1]\) with this structure that identifies an interval within \([0, 1]\) of allowable speeds. Policy space is now two-dimensional though case space remains one-dimensional. Typically, however, judicially announced policies are simple partitions in the sense they usually create two equivalence classes (see Kornhauser 1992).
join). Importantly, each justice’s pair of decisions must satisfy an endorsement-consistency constraint: if a justice joins the opinion, her disposition vote must conform to that entailed by the opinion. Formally, if \( s_i = 1 \) then \( d_i = r(\hat{x}, x) \).

The sequence of play is as follows:

1. A case arrives.
2. A writer \( j \) is designated, who writes an opinion.
3. Acting simultaneously, the non-writing justices first i) choose whether to join the opinion, and then ii) vote on the disposition of the case; the pair of actions must obey the endorsement-consistency constraint. Majority rule then determines the case disposition.
4. Non-authors receive payoffs based on their dispositional vote, join decision, the opinion’s content, and the case location. The author’s payoff is similar but also reflects the number of joins received by the opinion and whether a majority of the justices were in dissent (we discuss this possibility shortly).

//Insert Figure 1 about here //

Figure 1 displays the game form associated with the sequence of play, for a three member Court. Justice 1 is the opinion writer; opinions to one side of the case \( \hat{x} \) entail disposition 1, opinions on the other side entail disposition 2. As shown, Justice 2 makes a join decision and then casts a dispositional vote (e.g., “Disp1” or “D1” in the figure); simultaneously Justice 3 does the same (information sets are shown with dashed lines). The endorsement-consistency constraint makes some portions of the game tree unreachable. For clarity, we include these “ghost” portions in the figure but indicate them in gray. We assume the opinion author, Justice 1, joins his own opinion. Summary outcomes are shown at the terminal node using standard legal terminology.

A seemingly odd feature of the sequence of play is that the game may terminate with a majority of the justices dissenting from the author’s opinion. This is shown, for example, in the
bottom node of the top tree in Figure 1. However, as will be seen, this outcome is never an equilibrium in the game.\footnote{In practice, if a majority dissented the author would have to re-draft and re-submit his opinion. Thus, one can view the game form in Figure 1 as the stage game in an infinite horizon game, in which the game terminates in any stage in which a majority of the justices do not dissent. A seemingly natural solution concept would be stationary Markov perfect equilibrium; but here that implies the same equilibrium as in the stage game considered as a one-shot game. Hence, we focus on that game.}

The actions and sequence of play imply strategies in the game. An opinion-writing strategy for the author is $\chi_j$, a function from cases into rules, a proposed cut-point. That is, $\chi_j : \hat{X} \to X$. A join strategy is a function from cases and opinions into join decisions, $\sigma_i : \hat{X} \times X \to S$. A dispositional vote strategy is a function from cases, opinions, and own join actions into dispositions, $\delta_j : \hat{X} \times X \times S_i \to D$. An adjudication strategy for a non-writing justice is thus the ordered pair $(\delta_j, \sigma_i)$ while an adjudication strategy for the opinion author is the triple $(\chi_j, \delta_j, \sigma_j)$. However, in what follows, we require the opinion writer to join her own opinion.\footnote{As the author must write in any event, concurring with her own opinion simply requires her to write twice. In some, but not all instances, the opinion writer would rationally join her own opinion.} The endorsement consistency constraint then effectively reduces the opinion author’s strategy to the singleton, $\chi_j$, the opinion-writing strategy.

Outcomes follow from the players’ strategies. The disposition of the case results from simple majority rule applied to the nine dispositional votes. Call the majority winning disposition $\tilde{d}$. If $r(\hat{x}, x_j) = \tilde{d}$, the author’s opinion is compatible with the winning disposition – we call such an opinion a “majority-disposition compatible” opinion. The number of joins received by a majority-disposition compatible opinion plays an important role in the subsequent analysis. Define the
aggregate join function for opinion $x_j$ as $n(x_j) = \sum_{i \in J} s_i + 1$ (recall the author joins her own opinion). Finally, it is convenient to define the 9-tuple of disposition votes as $d \equiv (d_1, d_2, \ldots, d_9)$, the 9-tuple of join decisions as $s \equiv (s_1, s_2, \ldots, s_9)$, and the 9-tuple of join strategies as $\sigma \equiv \{\sigma_1, \sigma_2, \ldots, \sigma_9\}$.

Joins, Concurrences, and Dissents. We argue that join decisions and dispositional votes involve different considerations so it is important to consider adjudication strategies as the ordered pair $(\delta_j, \sigma_i)$. It is more common, however, to discuss the compound join-dispositional vote decisions; these compound actions have special names in legal terminology. For example, suppose the proposed opinion $x_j$ requires a ruling for the Plaintiff, given the facts in the case $\hat{x}$: $r(\hat{x}, x_j) = 1$ (recall equation (1)). Then the ordered pair of actions $(d_j, s_i) = (1,1)$ indicates a so-called “join”: a dispositional vote in accord with the content of the opinion and a join decision joining (endorsing) the opinion. The ordered pair of actions $(1,0)$ indicates a so-called “concurrence”: a dispositional vote in accord with the content of the opinion but a refusal to join (endorse) the opinion. The ordered pair of actions $(0,0)$ indicates a so-called “dissent”: a dispositional vote opposite to that indicated by the opinion and a refusal to join (endorse) the opinion.

Critically, the ordered pair of actions $(0,1)$ is not possible when $r(\hat{x}, x_j) = 1$ (and the $(1,1)$ pair is not possible when $r(\hat{x}, x_j) = 0$): in American jurisprudence a justice is not allowed to join the opinion yet cast a dispositional vote contrary to that required by the opinion’s rule when applied to the case. So, for example, a justice cannot endorse a rule that requires a disposition for the Defendant but then vote for a disposition in favor of the Plaintiff (simultaneously “join” and “dissent”). This is the endorsement-consistency constraint discussed previously: If $s_i = 1$ then $\delta_j(\hat{x}, x \mid s_i = 1) = r(\hat{x}, x)$. 

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3.3 UTILITY

3.3.1 UTILITY OF NON-AUTHORING JUSTICES

We define the utility of a non-writing justice as a function over her actions, given the case and the opinion: \( u_i : D \times S \times X \times \hat{X} \rightarrow \mathcal{R} \). Before we define this function, we require the following. First, let \( \bar{x}_i \) be justice \( i \)'s ideal rule, a point in \( X \). Note that justice \( i \)'s ideal disposition of the case is \( r(\hat{x}, \bar{x}_i) \) (using equation (1)). Second, define the indicator function

\[
I(d_i, \hat{x}, \bar{x}_i) = \begin{cases} 
1 & \text{if } d_i \neq r(\hat{x}, \bar{x}_i) \\
0 & \text{otherwise}
\end{cases}
\]

This function takes the value “1” if the justice’s actual disposition vote does not correspond to her ideal disposition of the case, and takes the value “0” if it does. Third, let \( k \) denote the effort cost of writing a concurrence or dissent, an explanation of why the author’s opinion is a poor rule (it is a norm in American jurisprudence that justices explain their actions).

We can now define non-writing justice \( i \)'s utility:

\[
u_i(d_i, s_i, x, \hat{x}) = s_i \nu(x_j, \bar{x}_i) - (1 - s_i)k - I(d_i, \hat{x}, \bar{x}_i)
\]

Equation (2) has the following interpretation. If the justice endorses the author’s opinion by joining it (so \( s_i = 1 \)), she receives a policy loss \( \nu(x_j, \bar{x}_i) \) through her association with the opinion. If she declines to join the opinion, she does not suffer this loss but she must pay the effort cost \( k \) required to write a concurrence or dissent. Finally, if her dispositional vote is not in accord with her ideal disposition of the case, she suffers a dispositional loss \( \gamma \). We require \( \gamma \geq 0 \).

We assume the policy loss function \( \nu(x_j; \bar{x}_i) \) attains a minimum loss at the ideal rule \( \bar{x}_i \), is continuous and involves increasing loss for opinions increasingly distant from the justice’s ideal rule, is symmetric around the justice’s ideal rule, and displays the single crossing property, as is standard in the spatial theory of voting. An example of such a loss function is the quadratic loss
function: $-(x_j - \bar{x}_j)^2$. Thus, we assume a justice prefers to be associated with a rule that more closely resembles her ideal rule.

The utility of non-writing justices is defined over all possible combinations of join choices and dispositional votes but the endorsement-consistency constraint precludes a simultaneous “join” and “dissent.” The endorsement-consistency constraint can lead to tension between casting the “correct” dispositional vote in the instant case and endorsing a relatively attractive opinion, a point discussed in detail in Section 4.

### 3.3.2 Utility of the Opinion Writer

We assume the opinion writer has preferences identical to those of the non-writing justices in all respects save two. First, the opinion writer cares not only about her dispositional value ($\gamma$) and association with a policy ($v(x_j; \bar{x}_j)$) but also about the “clarity of the law.” Specifically, we assume the opinion writer prefers a majority-disposition compatible opinion with more joins to the same majority-disposition compatible opinion with fewer joins. (Recall that a majority-disposition compatible opinion entails a disposition in the case that is the same as the majority winner in the dispositional vote). We introduce this aspect of her preferences in the simplest possible way: her preference for joins is separable from the other aspects of her preferences. Second, the opinion writer suffers a large loss from failing to author a majority-disposition compatible opinion. We thus have:

$$u_j(d_j, s_j = 1, x_j, \hat{x}) = \begin{cases} \beta n(x_j) + v(x_j, \bar{x}_j) - \gamma I(d_j, \hat{x}, \bar{x}_j) & \text{if } \tilde{d} = r(\hat{x}, x_j) \\ v(x_j, \bar{x}_j) - \gamma I(d_j, \hat{x}, \bar{x}_j) - \kappa & \text{otherwise} \end{cases} \quad (3)$$

The top component in (3) accrues to the opinion writer if her opinion is compatible with the majority-winning disposition; she receives the bottom component if it is not. The parameter $\beta$ indicates the marginal value to the author of an additional join when her opinion is majority-
disposition compatible; we assume $\beta \leq 1$. (Recall the aggregate join function for opinion $x_j$, $n(x_j)$).

We require $\kappa$ to be large enough so that penning the most attractive majority-disposition incompatible opinion is always worse for the author than penning the least attractive majority-disposition compatible opinion. We suppress the cost of writing $k$ for the opinion author as she is required to produce an opinion – her effort cost is infra-marginal.

3.3.3 DISCUSSION

Each of a justice's two actions – the disposition vote and the join decision – affects a distinct public good. The first public good is the majority-winning case disposition. The second is the degree of "clarity" or precedential value for a majority-disposition compatible opinion, which results from aggregating the join decisions. Precedential value may jump as the number of joins passes through five, as discussed earlier. To the extent the justices value these public goods, they must engage in extremely sophisticated calculations about the pivotality of their dispositional vote and the impact of their join decision on the precedential value of a disposition-majority compatible opinion. Strategic calculations about the two public goods may interact in complicated ways. For example, is it better to achieve a clear precedent even if doing so brings the wrong disposition in the instant case?

Our specification of utilities allows us to analyze a baseline case that abstracts from these public goods problems as far as possible: we assume a non-authoring judge evaluates her dispositional vote and join choice purely as acts in themselves; we assume the opinion author also

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9 The most attractive majority-disposition incompatible opinion will be written at the author’s ideal point and allow her to cast a “correct” dispositional vote. The least attractive majority-disposition compatible opinion will be written at the most distant location, gain no joins but her own, and require the author to vote for the “incorrect” disposition. Let $\bar{v} = v(0,1)$ and $\bar{v} = v(x_j, x_j)$. Then we require $\kappa \geq \bar{v} - v + \gamma - \beta$. 
evaluates her own actions but, as the “owner” of the opinion, also cares about its clarity or precedential value. The assumption of act-oriented justices follows some noted models of electoral competition which treat voters in a similar way (e.g., Callender & Wilson 2007, Hinich, Ledyard & Ordeshook 1972, Osborne and Slivinski 1993, Palfrey 1984). But arguably act-orientation is particularly appropriate for the judicial setting. It corresponds to the situation in which a judge asks herself, “What do I think is the right action here, in and of itself?” As will be seen, considerable strategic complexity emerges even in this baseline case.

4 EQUILIBRIUM

We now indicate sub-game perfect equilibria to the adjudication game. We proceed by backward induction. Hence, we begin with the dispositional vote strategies and then the join strategies of the non-writing justices. We then turn to the opinion author’s writing strategy. As the opinion author’s utility depends on the number of joins, we use the individual join strategies to define $n(x_j, \sigma)$, the aggregate join function given a vector of join strategies by the non-authors. We use this aggregate join function in tandem with the policy loss function and dispositional value to characterize the author’s writing strategy. The join and voting strategies of the non-authors and the author’s writing strategy together define an adjudication equilibrium.

4.1 VOTING AND JOINING STRATEGIES BY NON-AUTHORS

Given (2), the sequence of play, and the endorsement-consistency constraint, non-authoring justices have a simple dispositional voting strategy: they must vote for the disposition required by the opinion if they join the opinion, but if not they should vote so as to avoid a dispositional loss

\[ 10 \text{ It would be easy to allow non-writers to value clarity as well, provided clarity is linearly increasing in joins. Doing so in effect would reduce “k” for a non-writer.} \]
(that is, they should vote for their ideal disposition in the case). Recalling that \( r(\hat{x}, x_j) \) is the disposition required by the rule in the opinion given the facts in the case and \( r(\hat{x}, x_j) \) is justice \( i \)'s ideal disposition of the case in light of its facts, we have

\[
\delta(s_i, \hat{x}, x_j, \xi) = \begin{cases} 
    r(\hat{x}, x_j) & \text{if } s_i = 1 \\
    r(\hat{x}, \xi) & \text{if } s_i = 0
\end{cases}
\]  

(4)

Now consider the situation when the endorsement consistency constraint implies a dispositional loss, that is, when joining the opinion requires a dispositional vote other than the justice’s ideal disposition vote: this situation occurs when the cut-point in the opinion and the ideal cut-point of the justice lie on opposite sides of the case, that is, when \( \text{sgn}(x_j - \hat{x}) \neq \text{sgn}(\xi - \hat{x}) \). Call this an “opposite-side opinion” – joining an opposite-side opinion brings a dispositional loss. Conversely, the endorsement consistency constraint forces no dispositional loss in joining a “same-side opinion.”

From (2) and (4), it will be seen that joining versus non-joining involves a comparison between 1) a policy loss plus a dispositional loss (if the opinion is an opposite-side opinion) and 2) a writing cost. The policy loss will be less onerous when the opinion is not too distant from the justice’s ideal rule. Define the set of endurable opinions \( \Delta_i \)

\[
\Delta_i \equiv \begin{cases} 
    \{ x \mid v(x, \bar{x}_i) \leq k \} & \text{if } x \text{ is a "same-side opinion"} \\
    \{ x \mid v(x, \bar{x}_i) \leq k - y \} & \text{if } x \text{ is an "opposite-side opinion"}
\end{cases}
\]  

(5)

For example, if the policy loss function is a quadratic loss function, the set of endurable opinions is \( [\bar{x}_i - \sqrt{k}, \bar{x}_i + \sqrt{k}] \) for same-side opinions and \( [\bar{x}_i - \sqrt{k - y}, \bar{x}_i + \sqrt{k - y}] \) for opposite-side ones.

It is apparent, then, that a non-authoring justice should join an endurable opinion but no others:
\[\sigma_i(x_j, \hat{x}, \bar{x}, k, \gamma) = \begin{cases} 1 & \text{if } x_j \in \Delta_i \\ 0 & \text{otherwise} \end{cases} \quad (6)\]

Note that if a justice is indifferent between joining and not joining an opinion, (5) and (6) imply that she endorses the opinion.\(^{11}\)

We summarize the above analysis is the following proposition.

**Proposition 1** (Non-authors’ adjudication strategy). The adjudication strategy \((\delta_i, \sigma_i)\) for non-authoring justices is given by (4) and (6), where \(\Delta_i\) is defined in (5) and \(r(\hat{x}, x)\) is defined in (1).

*Proof*. From the above discussion, (4) clearly specifies an optimal dispositional voting strategy. Similarly, given (4), a non-authoring justice can do no better in her join choice than by following (6). Q.E.D.

// Insert Figure 2 about here //</br>

An implication of Proposition 1 is that each justice has a “join region” around her ideal rule: if the opinion lies within the join region, she joins it; otherwise, she does not. This is shown in Figure 2. To make matters concrete, suppose in Figure 2 that the justice’s policy loss function is a quadratic loss function. Then her join region is the interval \([\bar{x}_i - \sqrt{k - \gamma}, \bar{x}_i + \sqrt{k}]\) (assuming \(\hat{x} > \bar{x}_i - \sqrt{k - \gamma}\)). If the author’s opinion lies in the region \([\bar{x}_i - \sqrt{k - \gamma}, \hat{x}]\) the justice will join it even though the endorsement constraint forces her to vote for the “wrong” disposition. This occurs because the opinion is so attractive. We call this join behavior a strategic join or a *cross-over join*, since it involves endorsing an opposite-side opinion. If the dispositional loss \(\gamma\) is small or zero, the region in which the justice is willing to engage in a cross-over join expands. Conversely, if \(\gamma \geq k\) a

\(^{11}\) This specification avoids an open set problem in the author’s optimization problem.
judge will never engage in a cross-over join, so she is unwilling to join a highly proximate opinion if it yields the “incorrect” case disposition.

4.1.1 THE AGGREGATE JOIN FUNCTION

The aggregate join function \( n(x_j; \sigma) \) consists of the join from the opinion author \( j \), plus the sum of the join decisions of the non-writing justices as required by (6):

\[
n(x_j; \sigma_j) = 1 + \sum_{i \neq j} \sigma_i(x_j; \bar{x}_i, \hat{x}, k, \gamma)
\]

An illustrative aggregate join function is shown in the left-hand panel of Figure 3.

//Insert Figure 3 about here //

The aggregate join function’s exact shape depends sensitively on the distribution of ideal points, the cost of writing concurrences and dissents and – when the justices value correct case dispositions – the case location and the magnitude of dispositional losses. Broadly speaking, however, the aggregate join function takes the form of “steps” each indicating a specific number of joins in a segment of the case space. The aggregate join function is not continuous (though it is drawn so in Figure 3 for ease of visualization) but given the definition of the individual join functions it is upper semi-continuous, a fact of some importance subsequently.

4.2 AUTHORING STRATEGY BY OPINION AUTHOR

We now prove the existence of an optimal authoring strategy by the opinion author. Recall the opinion writer’s objective function (3):

\[
u_j(d_j, s_j = 1, x, \hat{x}) = \beta n(x_j, \sigma) + v(x_j; \bar{x}_j) - \gamma(d_j, \hat{x}, \bar{x}_j)
\]

The function \( n(x_j, \sigma) \) indicates the aggregate join function when the non-writing justices employ the equilibrium join strategies given by (6) (as before, \( v(x_j; \bar{x}_j) \) is the writer’s policy loss from rule \( x_j, \gamma \) is the dispositional loss the writer occurs from voting for an “incorrect” case disposition, and
the indicator function $I(d_j)$ indicates whether the opinion writer supports an incorrect case disposition). In words, the opinion writer wishes to set the content of her opinion so as to maximize the net gain from joins less the loss of departing from her most-preferred rule and any dispositional loss.

The following lemma asserts that the space of opinions over which the writer chooses is a compact set; a proof is in the Appendix.

**Lemma.** The set of opinions that command a dispositional majority, $X_d(\hat{x})$, is a compact set.

**Proposition Two** (existence). There exists an opinion $x_j^* \in X_d(\hat{x})$ that maximizes (2).

**Proof.** The aggregate join function is upper semi-continuous and the policy loss function is continuous on the entire case space, so their sum is upper semi-continuous on that space. From the lemma, the set of opinions that command a dispositional majority, $X_d(\hat{x})$, is a compact subset of the case space. Accordingly, from an extension to the extreme value theorem, (3) must achieve a maximum on $X_d(\hat{x})$ (see Theorem 2.43 in Aliprantis and Border 2005 (p. 44)). QED

In fact, it is easy to characterize $x_j^*$, at least in broad terms. Consider the step (or steps) of the aggregate join function whose range contains an argmax of (3). If the opinion writer’s ideal rule is also an element of that step’s domain and can command a dispositional majority, the writer offers her ideal policy as $x_j^*$. If her ideal rule is not in the domain of that step or cannot command a dispositional majority, she offers the element of the step’s domain closest to her ideal rule that can do so, the element on the edge step’s support in the direction of her ideal rule. Note that $x_j^*$ is always well defined, as every point that is a member of an open set for one step is a member of a closed set for a higher step.

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It is possible for more than one point to maximize (3), although this situation is clearly somewhat special. In such a case, a writer would be free to offer either the maximizing opinion that is closer to her ideal rule, or the maximizing opinion that attracts more joins.

5 EXPLORING THE MODEL

We now examine some of the model's implications, focusing on equilibrium opinions and the justices' voting behavior. We first consider a baseline example. We then examine the impact of changing the Court's personnel and the impact of changes in some of the model's key parameters.

5.1 BASELINE EXAMPLE

We begin with an example. In the example we assume policy losses are quadratic, the case does not present justices with a dispositional value \( \gamma = 0 \), and the writing cost \( k = .05 \), so that a justice will join an opinion if and only if it lies within \( \sqrt{k} = .22 \) of her ideal policy.

**Aggregate Join Functions in a Non-polarized Court.** Suppose the nine justices are quite non-polarized, so that justice 1 has an ideal point at .1, justice 2 at .2, and so on. The left-hand panel of Figure 3 shows the aggregate join functions facing Justice 2; the functions for the other justices are broadly similar but not identical, since the identity of the non-writing 8 justices varies. To aid visualization, we draw the aggregate join functions as continuous; in fact, they are only upper semi-continuous. The opinion author always joins her own opinion since she is obliged to pay \( k \) in any event, a sunk cost. Parts of the aggregate join function far from an opinion author reflect this single assured join.

**Opinion Locations.** The policy loss function and aggregate join function facing an opinion author are key components in her decision where to locate her opinion. But also important is the case location even when the dispositional value is negligible. This is because the opinion author is constrained to write a majority-disposition compatible opinion, one for which the number of joins
plus the number of concurrences in greater than five (or equivalently, the number of dissents is no
greater than four). For the moment, we assume an extreme case location (greater than .9 or less
than .1) to avoid this complication. For such an extreme case, there are no dissents (all non-joins
are concurrences) so the “majority disposition constraint” is immediately satisfied. We examine the
implications of a non-extreme case shortly.

In the baseline example, assume Justice 2 is the opinion author. As shown in the left-hand
panel of Figure 3, an opinion written at Justice 2’s ideal policy would garner four joins (including
her own). An opinion placed at several more central locations would gain six joins; of these
locations, the one closest to Justice 2 is the location that receives endorsements from Justices 3-7,
plus Justice 2. This location occurs at \( .7 - \sqrt{.05} = .48 \).

Suppose Justice 2 values joins at \( \beta = .06 \). Justice 2’s utility function is shown in the right-
hand panel of Figure 3. (To ease visualization, the utility function is drawn as a continuous function
but in fact it is only upper semi-continuous). The function attains a clear maximum, as indicated by
Proposition 2. In fact, it will be seen that the utility-maximizing opinion for Justice 2 is the closest
opinion that gains five joins, that is, joins from a coalition of Justices 1-5. As Justice 5 is the most
distant member of this coalition, the optimum opinion is located at the nearer edge of Justice 5’s
acceptance region: \( .5 - \sqrt{.05} = .28 \). If Justice 2 valued joins somewhat more highly, she would
location her opinion at the nearest join maximizing location, .48, thereby receiving six joins. If she
valued joins somewhat less, she would offer a policy at her ideal point, .2, gaining four joins (those
from Justices 1-4). In the latter case, the case location must also be such that at least one additional
justice concurs with the disposition implied by the opinion, if the case location lay at or above
Justice 5’s ideal policy.
Effect of Opinion Assignment on Opinion Location. As the previous example suggests, in the adjudication model opinion assignment can be extremely consequential. Figure 4 indicates the optimum opinion for each justice in the non-polarized Court with extreme case location.

// Insert Figure 4 about here //

As shown, Justices 3-7 author at their ideal policy. Because they are centrally located in the non-polarized court, they need not deviate from their most preferred rule to garner joins. Justices 1, 2, 8 and 9, however, locate opinions more centrally than their ideal policy, seeking joins. Notably, the most extreme justices, 1 and 9, locate their opinions more centrally than their slightly less extreme neighbors, Justices 2 and 8. This results from the “gravitational pull” of Justices 1 and 9 on Justices 2 and 8. Figure 4 underscores the irrelevance of the median voter theorem for opinion content in the model, though of course the preferences of the median voter are extremely consequential for case dispositions.

Disposition-Majority Compatibility. Suppose the case location were not extreme but in fact rather central, say, \( \hat{x} = .55 \), so the ideal policies of justices 1-5 lie to the left of the case location and those of justices 6-9 lie to the right. Non-joins may be either concurrences or dissents, depending on whether the voting justice’s ideal policy is on the same side or the opposite side of the case as the opinion. As a result, the opinion writer may be constrained in locating her opinion by the need to hold dissents below five.

//Insert Figure 5 about here //

Figure 5 shows the aggregate join functions and aggregate dissent functions facing Justice 9 in the example. The functions indicate the number of joins and the number of dissents at each case location for cases authored by this justice. As shown, an opinion located far to the right (above .72, the vertical line in the figure) would provoke 5 dissents so the opinion would not be disposition-majority compatible. Because of the resulting loss to the author (recall (3)) Justice 9 would not
locate opinions on the far right of the policy space, above .72. In fact, though, under the assumed parameter values ($\beta = .06$) Justice 9 prefers to locate her opinion somewhat more centrally in order to gain more joins so that disposition-majority compatibility does not enter her calculations.

Figure 6 shows the dispositional vote associated with each justice’s optimal policy under the assumed parameters, that is, the dispositional votes generated by the opinions indicated in the author-opinion diagram, Figure 4. The dispositional votes range from 7-2 to 5-4. A striking feature of the adjudication model is that not only do opinion locations vary with the opinion author; so can case dispositions. Given a central case location, opinion authors on the left side of the Court craft opinions that draw support from a center-left coalition in favor of one disposition; authors on the right side of the Court craft opinions that draw support from a center-right coalition in favor of the other disposition. This feature of the model underscores the importance of opinion assignment, and suggests the importance for case dispositions of the Chief Justice’s assignments.

//Insert Figure 6 about here //

Connected Join-Coalitions. In the baseline example, the join-coalitions are “connected” but they need not be. More specifically, let $x_l$ be the leftmost member of a join-coalition and $x_r$ the right most member of a join-coalition. Call the coalition that joins an opinion connected if every justice with ideal point in the interval $[x_l, x_r]$ is also in the coalition. In the model, join-coalitions need not be connected because the opinion author may be somewhat separated from the other justices who join the author’s opinion. Excluding the opinion author, however, join-coalitions should be connected. Of course, depending on the opinion location, the same number of joins may occur with quite different join-coalitions. And, the coalition concurring with the disposition supported by the opinion – the concurrence-coalition – need not be connected.

5.2 CHANGING THE COURT’S PERSONNEL
Consider a Court in which Justice $i$ is an opinion author. What happens when Justice $i$ is replaced by a new Justice $j$ with different preferences? Does the content of the opinions written by $j$ differ from those written by $i$? Call this the “direct effect” of a nomination (Cameron et al 2009). What about the content of opinions written by the other justices? Call this the “peer effect” of a nomination (ibid). The median voter framework, for example, predicts zero direct and peer effects unless the change alters the location of the median. Given a change in the location of the median, the median voter framework predicts uniform peer effects. Similarly, the majority median framework predicts zero direct and peer effects, conditional on the same disposition-majority median.

**Proposition 3** (Direct Effect of Nominations). Fixing the remainder of the Court, as the ideal rule of the opinion writer becomes more conservative but the writer’s preferences about the case disposition remain unchanged, the opinion’s content moves (weakly) in a conservative fashion if and only if the policy loss function displays increasing differences in the writer’s ideal rule.

*Proof.* The parameter of interest $\bar{x}_j$ enters the policy loss function and the “correct disposition” indicator function. But we stipulate that the I(d) function remains fixed. Using Proposition A1, in this case increasing differences for (2) requires $\frac{\partial^2 v(x_j; \bar{x}_j)}{\partial x_j \partial \bar{x}_j} > 0$, that is, the policy loss function must display increasing differences. QED

**Example.** Suppose the policy loss function is the quadratic loss function $- (x_j - \bar{x}_j)^2$. Then $\frac{\partial^2 v(x_j; \bar{x}_j)}{\partial x_j \partial \bar{x}_j} = 2$ and opinion content weakly increases when the writer becomes more conservative (but does not thereby alter his preference about the case disposition).

Proposition 3 verifies that the adjudication game is an author-influence model when the writer’s policy loss function displays increasing differences, as is commonly assumed in the spatial theory of voting. The Proposition also indicates that, if the adjudication model is a reasonable
representation of the operation of the Supreme Court, nominations are not a "move-the-median" game as is often assumed in formal models of nomination politics (Krehbiel 2007, Moraski and Shipan 1999, Rohde and Shepsle 2007). Rather, each nominee is potentially consequential for the Court's policy, which is often not true in move-the-median games.

A simple example illustrates non-monotonic peer effects: a justice's opinions may move left (say) even though the new justice is farther right than justice she replaces. For example, consider a Court with three justices on the far left, two slightly left of center, and four on the far right, with the opinion writer being one of the centrists. If the writer only moderately values joins, she will place the opinion at her own ideal point, receiving two joins. Suppose that the other centrist then departs and is replaced by another justice on the far right, thus shifting the median justice to the far right. The median voter approach requires the centrist author to shift her opinion to the right as well. But in fact in the model presented here, the centrist author may shift left, seeking joins from the more proximate block.\textsuperscript{12} Non-monotonic peer effects appear to be a distinctive prediction of the adjudication model.

5.3 VALUE OF JOINS (LEGAL CLARITY) TO THE OPINION WRITER (\( \beta \))

Because the ultimate impact of a legal rule depends on the actions of other players such as potential litigants and lower court judges, it can be important for the Court to speak with a unified, powerful voice. In such circumstances the opinion writer will particularly value joins and, conceivably, may alter the content of his opinion in order to gain them. In the model, increased value of legal clarity

\textsuperscript{12} More specifically, assume Justices 1-3 have ideal rules at 0, Justices 4 and 5 at \( A \), and Justices 6-9 at .95, with \( k = .04 \), \( \beta = .015 \), and an extreme case location. Then Justice 4 initially places her opinion at \( A \); but if Justice 5 departs and is replaced by another justice at .95, Justice 4 moves leftward her opinion to .2. Numeric calculations for this example are available from the authors.
corresponds to a larger value of the parameter $\beta$. The following result addresses changes in the value of joins.

**Proposition 4** (Change in Value of Joins). When the opinion writer places a greater value on attracting joins, the opinion’s content moves (weakly) in the direction of gaining additional joins.

**Proof.** In this case, the parameter of interest, $\beta$, affects only the aggregate join function. Consequently equation (A1) becomes

$$\beta^H n(x_j^H) - \beta^L n(x_j^L) > \beta^L n(x_j^H) - \beta^L n(x_j^L, y^L)$$

$$\Rightarrow \beta^H [n(x_j^H) - n(x_j^L)] > \beta^L [n(x_j^H) - n(x_j^L, y^L)]$$

Which is true provided $n(x_j^H) > n(x_j^L)$. In other words, $x_j^*$ weakly increases if doing so brings additional joins. QED.

This result stands in stark contrast with median voter and majority median approaches, which predict that every case has the same content irrespective of the importance to the opinion writer of the Court speaking with a unified voice.

**Join Maximizing $\beta$.** For $\beta$ sufficiently large, the importance of joins to the writer outweighs the loss from announcing a policy far from her ideal policy. Thus, for $\beta$ sufficiently large, the opinion writer seeks to maximize the number of joins.

**Small Value of Joins.** Consider now the converse case when $\beta$ is very small. Here clarity in the law has little value to the opinion writer. When $\beta$ is sufficiently small, an author will write at her ideal point so that an “author monopoly” result occurs.

5.4 DISPOSITIONAL VALUE $\gamma$
Suppose a case presents the justices with dispositional value so that voting for an "incorrect" disposition is somewhat, or even very, unpleasant. In the model, this situation corresponds to $\gamma > 0$.

First, consider behavior when $\gamma > k$. In this case, for a non-writing justice, the set of endorseable opinions $\Delta_i$ is restricted to the same-side opinions. Therefore, a non-writer prefers to dissent rather than cross-join or concur in an opposite-side opinion. This parameter value thus yields a situation similar to that in Carruba et al 2008, in which no judge votes strategically on the disposition. This situation emerges as a limit case in the model considered here.

What happens to opinion locations as dispositional values change? To answer this question, we must first consider how the aggregate join function changes as dispositional values change. As $\gamma$ decreases ($0 \leq \gamma < k$), justices on the "far" side of the case are increasingly willing to join a "near" side opinion if its location is proximate, even though it yields the incorrect disposition from their perspective. Consequently, for a justice on the "near" side of the case a portion of the aggregate join function shifts upward near the case location.

Example. Consider the non-polarized court with case location $x = .55$ and cost of writing $k = .05$. In this case, the left-most justice above the case is Justice 6 with $x_i = .6$. Given quadratic policy loss, the left-most edge of Justice 6's acceptance region extends below the case as far as $x = .6 - \sqrt{k - \gamma} = .376$ when $\gamma = 0$ but only to the case itself when $\gamma \geq .0475$. Figure 7 shows the baseline aggregate join function at $\gamma = 0$ (solid line), $\gamma = .03$ (small dashing), and $\gamma = .05$. This join function is the same below .376 for all three dispositional value; this part of the join function is shown as a heavy solid line. Above .376 the join function for $\gamma = .05$ is show with a thin solid line. When $\gamma = .03$ the join function shifts up by one above $x = .6 - \sqrt{k - \gamma} = .46$ as Justice 6 will cross-join such opinions (shown with small dashing). But the function shifts up by no more than one join since
.7 - \sqrt{k - \gamma} = .56$, that is, Justice 7 does not cross-join opinions below the case location. When $\gamma = 0$ the function shifts up by one (relative to $\gamma = .05$) in the region $0.376 - 0.476$ as Justice 6 cross-joins such opinions, but by two joins in the region $0.476 - 0.55$ as both Justice 6 and Justice 7 cross-join such opinions.

//Insert Figure 7 about here //

The monotonicity of additional joins in the example is not an accident; it is necessarily implied by the geometry of the acceptance regions. That is, not only is no mass added to the join function farther than the edge of the acceptance region of the most proximate justice on the opposite side of the case; the number of new joins added to the join function on the “near” side must weakly increase with proximity to the case location. This monotonicity allows the derivation of a comparative static result. Let $k \geq \gamma_H > \gamma_L \geq 0$ and denote the equilibrium opinion location associated with $\gamma_H$ as $x_j(\gamma_H)$ and that associated with $\gamma_L$ as $x_j(\gamma_L)$.

**Proposition 5** (Change in Dispositional Value). A) If the opinion writer is a join maximizer and the maximum lies on the same side of the case as his ideal point, then if $\gamma_H$ decreases to $\gamma_L$, $x_j(\gamma_L)$ moves weakly in the direction of the case location (relative to $x_j(\gamma_H)$). B) If $x_j(\gamma_H)$ lies farther from the case than the region in which the aggregate join function increases when $\gamma_H$ decreases to $\gamma_L$, then $x_j(\gamma_L)$ moves weakly in the direction of the case (relative to $x_j(\gamma_H)$).

*Proof.* Part A is a consequence of the monotonicity of change in the join function discussed above. By assumption, $x_j(\gamma_H)$ is associated with a maximum in the join function. As dispositional value decreases, more mass cannot be added to the join function at any location farther from the case location than $x_j(\gamma_H)$ than is added to all locations closer to the case than $x_j(\gamma_H)$. Consequently, a new join maximum cannot be created farther away from the case than $x_j(\gamma_H)$ (although a new
maximum can be created closer to the case location). Hence, the new equilibrium opinion location must remain in the same location or shift in the direction of the case location; it cannot shift farther away from the case location. B) By assumption \( x_j(\gamma_H) \) lies in a region such than no new mass is added to the join function farther from the case location than \( x_j(\gamma_H) \); consequently, \( x_j(\gamma_L) \) cannot move farther from the case location than \( x_j(\gamma_H) \). Q.E.D.

Proposition 5 is silent about the situation when a non-join maximizer writes an opinion \( x_j(\gamma_H) \) that lies in the region in which the join function increases when \( \gamma_H \) decreases to \( \gamma_L \). In this situation, additional mass is added to the join function on both sides of the original opinion location and the writer may, under different circumstances, move the opinion in either direction.

### 5.5 CASE LOCATION \( \hat{x} \)

If the case location \( \hat{x} \) lies either to the right of the ideal point of the most conservative justice or to the left of the ideal point of the most liberal justice, it induces unanimity with respect to the disposition of the case. Of course, the Court, though unanimous about the case disposition, may display extreme disagreement about the best location of the opinion. Cases more interior may induce disagreement about the case disposition as well.

**Example.** Consider again the baseline example, with \( k = .05 \) and \( \gamma > k \). In this situation there are no cross-over joins. What happens to the aggregate join function as the case location shifts? This is shown in Figure 8, in which the case location increases gradually from \( \hat{x} = .55 \) to \( \hat{x} = .95 \).

//Insert Figure 8 about here //

As the case moves from left to right, some justices may continue to join the higher case. In addition, some justices formerly to the right of the case may join those already on the left of the
case. These “uncovered” justices will join an opinion to the left of the case location, if the opinion is sufficiently proximate. As a consequence, the effect of the shift is to 1) extend the upper edge of the aggregate join functions for justices to the left of the case and 2) add mass to the aggregate join functions for justices to the left of the case. This mass must lie above the lower edge of the acceptance region of the most leftward of the newly “uncovered” justices. For instance, in the example suppose the case moves from .55 to .75. The upper edge of the acceptance region of Justice 4 now extends not to .55 (the old case location) but to .624 and that of Justice 5 extends not to .55 but to .724. So the base of joins from the old justices to the left of the case shifts upward from zero to two in the region [.55, .624] and one in the region [.624, .724]. In addition, Justices 6 and 7 are “uncovered.” The acceptance region of Justice 6 for opinions to the left of the case location is \(0.6 - \sqrt{0.05} = 0.376\) while that of Justice 7 is \(0.7 - \sqrt{0.05} = 0.476\). Hence, the indicated aggregate join function further shifts upward by one join in the region \([0.376, 0.476]\) and by two joins in the region \([0.476, 0.75]\).

Consider \(\hat{x}_1 < \hat{x}_2\). For justice \(i\) define \(\min(\Delta_i(\hat{x}_1))\), the minimum endorsable opinion given \(\hat{x}_1\); similarly, let \(\max(\Delta_i(\hat{x}_1))\) be the maximum endorsable opinion given \(\hat{x}_1\). A critical feature of a change in case location from \(\hat{x}_1\) to \(\hat{x}_2\) is that aggregate join functions for justices below \(\hat{x}_1\) take on additional mass only above \(\min(\Delta_i(\hat{x}_1))\), for example, above \(\hat{x}_1 - \sqrt{k - \gamma}\) in the baseline example.

**Proposition 6** (Change in Case Location). If \(x_j(\hat{x}_1)\) lies below \(\min(\Delta_i(\hat{x}_1))\) then as \(\hat{x}_1\) increases to \(\hat{x}_2\), \(x_j(\hat{x}_2)\) moves weakly upward. Conversely, if \(x_j(\hat{x}_1)\) lies above \(\max(\Delta_i(\hat{x}_1))\) then as \(\hat{x}_1\) decreases to \(\hat{x}_2\), \(x_j(\hat{x}_2)\) moves weakly downward.

**Proof.** The condition assures that, relative to the original opinion location, mass of the justice’s aggregate join function increases in one direction while remaining unchanged in the other. The
policy loss function and dispositional value remain unchanged. Accordingly, the new opinion location cannot shift in the direction away from the mass gain. QED.

The proposition indicates that an opinion written by a relatively extreme justice (relative to the case location) moves weakly in the direction of the case location, as the location shifts in a more extreme direction. The proposition is silent about what happens when a moderate justice is the author.

6 CONCLUSION

Models of collegial courts should take adjudication seriously. By this we mean, models of collegial courts should reflect the institutional features that distinguish adjudication from legislative and electoral contests, especially if those features seem likely to shape adjudicatory outcomes in a profound way. We have highlighted three features of adjudication in particular:

- First, *adjudication jointly determines a case disposition and “policy” or opinion content*. Although case dispositions must be compatible with opinion content, the linkage between the two is complex. For example, a majority, indeed a unanimous, view on case disposition does not imply majority (or unanimous) agreement on policy. In fact, in the United States, adjudicatory institutions do not require that any opinion receive majority support. Moreover, the joint production of case dispositions and policy outcomes may present a judge with difficult tradeoffs between the better case disposition and the best rule on offer.

- Second, *the rules used to choose policies are radically different from those used in legislatures or elections*. There is no amendment agenda. There is no paired comparison of alternatives. There is no vote pitting alternatives against the status quo. There is no run-off between tied alternatives. Voters may abstain from endorsing any opinion, and doing so is consequential for the clarity of the Court’s voice.
Third, in light of the procedures employed to determine outcomes, *judicial motivations are apt to be critical in determining case dispositions and policy.* Doing the right thing, announcing the best rule, and achieving clarity in the law may be as or more important than triumphing over competitors.

We have taken some tentative steps towards incorporating these features in a formal, game theoretic model of adjudication. First, the model incorporates joint production of case dispositions and policy outcomes and links the two. Second, the model incorporates some of the distinctive procedures used to choose policy. For example, we model joins, dissents, and concurrences. Third, we posit distinctively judicial preferences, including preferences for case dispositions and preferences for policy, and desires by opinion writers to achieve clarity in the law. We have not considered free entry of opinions nor competition between potential majority opinions, though we believe the model can be extended to address them. We believe most of the reported results will remain.

The model predicts behavior that is clearly at odds with the median justice model and the “median of the majority” model. For many parameter values, the location of an opinion depends critically on the entire distribution of ideal points, not simply on the location of the median justice. Dense clusters of ideal points tend to exert a “gravitational pull” on opinions. In courts with a strong center, outcomes may resemble median outcomes. In general, though, the model is an “author influence” model and for some parameter values is a “monopoly author” model. As such, the model suggests the importance of opinion assignment and strategic calculations by the Chief Justice. Other critical parameters affecting opinion locations and join decisions include the costs of writing separately and judicial valuations of a correct disposition and legal “clarity.”

Theory that takes adjudication seriously has significant implications for empirical work. One obvious example is the literature scaling justices’ “ideal points” using dispositional votes. The
model here indicates that such scaling methods are, at a minimum, highly inefficient because they discard the information in join decisions. More ambitiously, efficient use of all the information in the joint disposition-join decision may allow scaling not only of ideal points but opinion locations and case locations. Such information could be extremely valuable in assessing theories of collegial courts, as well as understanding the substance of Supreme Court policy making.

**APPENDIX**

**PROOF OF LEMMA**

The lemma asserts that the set of opinions that command a dispositional majority, \( X_d(\hat{x}) = \{ x \mid n(\hat{x}, x, j) \geq 5 \} \), is a compact set. The idea of the proof is that \( X_d(\hat{x}) \) may be decomposed into two types of opinions that yield dispositional majorities: 1) the set \( X_s(\hat{x}) \) of opinions that yield the “sincere” majority disposition (the disposition that would result if each justice voted for the disposition dictated by her ideal point), and 2) the set \( X_m(\hat{x}) \) of opinions that yield an “insincere” majority (a majority that can only occur if there are cross-over joins) (this set may be empty). The proof shows that each of these set of opinions is a compact set; consequently the union of the two sets is a compact set (a well-known result in topology).

Define the sincere case disposition correspondence:

\[
d_s(\hat{x}, \bar{x}_1, \ldots, \bar{x}_g) = \begin{cases} 
1 & \text{if } \sum_i (1) \text{sgn}(\hat{x} - \bar{x}_i) > 0 \\
0 & \text{if } \sum_i (1) \text{sgn}(\hat{x} - \bar{x}_i) < 0 \\
0 \text{ and } 1 & \text{if } \sum_i (1) \text{sgn}(\hat{x} - \bar{x}_i) = 0 
\end{cases}
\]

Lemma. \( X_d(\hat{x}) \) is compact.
Proof. First consider the multi-valued portion of the sincere case disposition correspondence, which occurs only when \( \hat{x} \) is located on the ideal point of the median justice. In such an instance, an opinion at any location in the case space must command a (sincere) dispositional majority even absent cross-over joins. So the entire case space corresponds to \( X_d(\hat{x}) \), which is therefore compact since \( X \) is the unit interval.

Now consider the single-valued portion. First consider \( X_s(\hat{x}) \), the members of \( X_d(\hat{x}) \) such that \( r(\hat{x}, x) = d_s(\cdot) \) (majority opinions whose disposition corresponds to the single sincere disposition). This set is either \([0, \hat{x}]\) or \([\hat{x}, 1]\); in either case, the set is compact.

Now consider \( X_{ns}(\hat{x}) \), the members, if any, of \( X_d(\hat{x}) \) such that \( r(\hat{x}, x) \neq d_s(\cdot) \). If the set is empty, it is compact. If the set is not empty, then there must be enough cross-over joins from the “insincere” side of \( \hat{x} \) to gain a majority. Consider a location on the insincere side such that a justice at this location is indifferent between joining and not joining the opinion. If this point lies outside \( X \) then \( X_{ns}(\hat{x}) \) is compact since it runs from \( \hat{x} \) to the boundary of \( X \) on the insincere side. If the indifference point is interior to \( X \), equations (5) and (6) require an indifferent justice to join, hence the set of insincere majority opinions is the closed interval from \( \hat{x} \) to the indifference point, a compact set.

This exhausts the possibilities. Thus, both \( X_s(\hat{x}) \) and \( X_{ns}(\hat{x}) \) are compact so their union, \( X_d(\hat{x}) \), is compact. Q.E.D.

**MONOTONE COMPARATIVE STATICS IN THE ADJUDICATION MODEL**
The following proposition provides the basic result governing comparative statics in the adjudication game.
**Proposition A-1.** (monotone comparative statics). $x_j^*$ is non-decreasing in a parameter if and only if (3) has increasing differences in the parameter. If the parameter enters only one of the components of (3) (e.g., the weighted aggregate join function or the policy loss function), $x_j^*$ is non-decreasing in the parameter if and only if that component of (3) has increasing differences in the parameter.

*Proof.* Follows from Theorem 2.3 in Vives (2000); see also Athey et al Theorem 2.3. Q.E.D.

The comparative statics of the model thus turn on demonstrating increasing differences in the parameter of interest. More precisely, where $x_j^{H} > x_j^{L}$ and parameter $y^{H} > y^{L}$, we require

$$u(x_j^{H};y^{H}) - u(x_j^{L};y^{L}) > u(x_j^{H};y^{L}) - u(x_j^{L};y^{L})$$  (A1)

This condition must often be checked directly rather than through the relevant cross-partial derivative $\partial^2 u(x_j;y)/\partial x_j \partial y$, which may not exist since the aggregate join function is not differentiable.

**REFERENCES**


Figure 1. Game Form for a Three Member Court.
Figure 2. An individual join function. Shown is the probability of joining an opinion with content $x_j$. For opinions outside the join region, the justice writes a concurrence or dissent expressing her preferred case disposition. Note the possibility of a strategic join: the justice will join some opinions to the left of the case location $\hat{x}$, implying an incorrect disposition from her perspective. Nonetheless she joins because the content of the opinion is so attractive.
Figure 3. Aggregate Join Function and Utility Function for Justice 2. Assumes a non-polarized Court. ($k = .05, \gamma = 0$).

Figure 4. The Author-Opinion Diagram for the Example. Different justices author quite different opinions. The dashed line is the 45-degree line. ($k = .05, \gamma = 0, \beta = .06$, extreme case location)
Figure 5. Constraints on Opinion Location Imposed by Disposition-Majority Compatibility. Shown are the aggregate join and dissent functions facing Justice 9 in the example. Joins are shown via the solid line, dissents by the dashed line. The omitted category is concurrences. The case is located at $\hat{x} = .55$, the cost of writing $k = .05$, and the dispositional value $\gamma = 0$. Justice 9 cannot locate opinions to the right of .72 (the vertical line) as doing so would fail to gain enough joins and concurrences to support the required case disposition.
Figure 6. Disposition Votes for the Justices' Optimal Opinions. The case location is shown by the vertical line. Given the parameters, justices to the left of the case write opinions yielding one disposition, those to the right write opinions yielding the other disposition. For example, Justice 1's opinion generates a 6-3 vote for Disposition One; Justice 8's opinion generates a 5-4 vote for the other disposition. ($k = 0.05, \gamma = 0, \beta = 0.06, \hat{x} = 0.55$)
Figure 7. Changing Aggregate Join Function as Dispositional Value Changes. As the dispositional value $\gamma$ decrease, justices on the “far” side of the case are more willing to engage in cross-over joins. As a result, the aggregate join function shifts upward on the “near” side of the case in a region close to case. Farther from the case, the aggregate join function is unaffected (thick solid dark line in figure). In the figure, the thin solid line corresponds to $\gamma = .05$, the dotted line to $\gamma = .03$, and the dashed line to $\gamma = 0$. 
Figure 8. Shifts in the aggregate join function as the case location shifts. As the case location moves upward, more justices find themselves on the “near” side of the case and become willing to join opinions on that side of the case. Accordingly, the aggregate join function shifts upward above the old case location. In the figure, the dispositional value is greater than the writing cost ($k = .05$) so there are no crossover joins.