Comparisons of the Incentive for Insolvency under Different Legal Regimes

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Comparisons of the Incentive for Insolvency under Different Legal Regimes

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Abstract

This paper compares, in the context of hazardous waste generation, the effects of joint and several liability on capital and production decisions to the effects of several only liability. Our main result shows that increased potential liability causes firms to decrease asset exposure, but may also lead firms to create less waste. First, we find that both several only and joint and several liability induce firms to go bankrupt more often and create more waste than is socially optimal. Then we find that, for a given level of funds, joint and several liability induces firms to go bankrupt more often and create more waste than several only liability. This implies that society will be responsible for a larger share of cleanup under joint and several liability than under several only liability. Finally, we show that firms with potentially higher liabilities for cleanup will raise less funds, creating “smaller” firms, and thus, the possibility of less waste generated overall.

1 Introduction

Two different legal regimes are commonly used to govern situations with multiple tortfeasors: joint and several liability, under which solvent firms are financially responsible for the liability attributed to insolvent firms; and several only liability, under which each firm is responsible only for its apportioned share of the liability. In their analysis of liability and negligence rules, Kornhauser and Revesz (1990) find that when firms have predetermined “exogenous” solvencies, neither rule dominates the other in terms of social welfare.¹

¹Throughout this paper, we define social welfare to be the sum of the benefits of generating waste for the firms, less the cost to the environment of this generation.
The assumption of fixed solvencies, however, is problematic. In response to the legal regime, firms may alter their solvencies to avoid future liability. The solvencies that firms choose, then, are “endogenous” to the legal regime. Furthermore, this choice of solvency may also affect the likelihood that the firm will disappear, and liability will not be assessed against the firm.

This paper extends the Kornhauser and Revesz (1990) model by incorporating an endogenous solvency and an endogenous probability of insolvency. Though our analysis applies to any situation with multiple tortfeasors, we present our argument in the context of the problem of disposal of hazardous waste by many firms at a single site. Two reasons justify this expository choice. First, Kornhauser and Revesz developed their argument in this context. Second, the presence of potentially insolvent actors at hazardous waste sites poses a significant problem of public policy that has occasioned much controversy. At each renewal of the Comprehensive Environmental Response, Compensation, and Liability Act (CERCLA), industry seeks to change the governing liability rule from joint and several to several only liability. Though the prior literature advanced the policy debate concerning the choice between these two rules, it remains inadequate as a basis for policy formulation precisely because it treats solvency as exogenous. In the hazardous waste context, liabilities are often large and they occur long after the profits have been realized. Firms have a clear incentive to reduce their solvencies to avoid liability. Our model provides some insight into the relative incentives created by the two liability regimes. We find that, while firms may generate less waste under a joint and several liability rule, firms become insolvent more often. Therefore, the policy maker must balance the potential environmental gains from a joint and several liability regime against an increase in the governmental share of unfunded liability.

Our model explores this trade-off. We consider two waste-generating firms that deposit waste at a site. Each chooses its solvency level and the amount of waste it generates. The choice of a lower level of solvency has countervailing effects on a firm: though "deep pockets" increase profits, they also imply an increased probability of solvency when liability is incurred; therefore, under joint and several liability, a firm with higher solvency is more likely to pay the share of liability attributable to insolvent firms. Our main result formalizes this intuition. We show that an increase in potential liability causes firms both to decrease asset exposure, and possibly to generate less waste. Our argument has several steps. First, we show that both several only and joint and several liability induce firms to go bankrupt more often, to underinvest in capital and to generate more waste than would be socially optimal. Second, we show that, for given levels of solvency, joint and several liability induces firms to go bankrupt more often and create more waste than several only liability. This implies that society will be responsible for a larger share of cleanup under joint and several liability than under several only liability. Finally, we show that firms with potentially higher liabilities for cleanup will raise less

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2 We use the words bankruptcy and insolvency interchangeably throughout the paper.
3 We do not address some policy issues. We do not address the important (and controversial) policy issue concerning retrospective liability, that is, the imposition of liability on firms for the deposit of waste prior to the enactment of the statute in 1980. Also, we ignore some complicated questions of bankruptcy law.
funds, creating “smaller” firms, and thus, the possibility of less waste generated overall.

The policy consequences of our argument are ambiguous. We cannot conclude that either rule dominates the other in terms of social welfare. Our model, however, highlights the factors that policymakers ought to consider when choosing between rules of joint and several liability rules of several only liability. Our results, moreover, provide an important corrective to the analysis in Kornhauser and Revesz [1990]. That paper suggests that the policymaker compare firms of equal solvency when choosing between joint and several liability and several-only liability. Our paper, however, indicates that this comparison inappropriately favors joint and several liability because the choice of liability regime affects not only the level of waste generation but also the solvency of the firm.

The paper first presents a brief literature review in section 2. Section 3 provides an example. Section 4 develops the model. Section 5 presents the baseline social welfare maximizing case. Section 6 investigates the problem with possible insolvency. Section 8 concludes with possible avenues for further research.

2 Literature Review

Under Superfund, all firms that deposit waste into a particular site are jointly and severally liable for the loss to society it causes, unless the firms can prove that the harm is “divisible,” or uniquely attributable to one party or another. Kornhauser and Revesz (1990) develop the basic two-player non-cooperative model for waste disposal with the possibility of insolvency. They find that neither several only liability nor joint and several liability dominates the other from a social welfare perspective, which they define as the sum of the benefits from the firms that produce waste, less the cost to society of waste generation. Two other papers, Yahya (2000) and Watts (1998) build on Kornhauser and Revesz (1990) and use a non-cooperative game framework. Watts (1998) examines this problem from the perspective of a Cournot quantity game. She also incorporates an exogenous probability of insolvency, which is not affected by the strategies the firms choose. She finds that neither liability rule dominates the other in terms of social welfare.

Yahya (2000) endogenizes the solvency condition. However, he approaches the problem differently. Specifically, he treats the solvency decision as separate from the waste generation decision; i.e., he models the optimal debt-equity ratio decision of a firm under the possibility that an accident may occur. More debt induces less liability; however, more debt increases the possibility of bankruptcy and ensuing bankruptcy costs. Therefore, while he endogenizes the solvency condition, he does not explicitly connect the solvency condition and waste generation decision. In that sense, then, he does not examine solvency as part of the production process per se, he merely looks at it in terms of the financial structure of the firm. In our model, solvency has real benefits, in terms of how the firm behaves in its production process.
Hansen and Thomas (1999) address a different yet related aspect of the hazardous waste liability problem. They explore the effects of shared liability between the owner of the waste site and the generator of the waste. They find that a shared liability rule may result in a more efficient outcome than a rule where only the owner of the waste site is held liable. This is because the owners of the waste site that take the least amount of care will also submit the lowest bid for accepting the waste. By sharing the liability between the generator and the owner of the waste site, the adverse selection problem is minimized.

Prior to Kornhauser and Revesz (1990), research on the effects of insolvency was not generally integrated with research concerning multiple tortfeasors. Shavell (1987), summarizes the effects of bankruptcy on care and the effects of multiple tortfeasors on care, but does not combine the two. Shavell summarizes by stating that “care” (in the framework used here, negative waste) is increasing in the assets of a firm.4 Beard (1990) also analyzes a one-person, optimal care game with the possibility of bankruptcy, and shows that Shavell’s conclusions may not be robust to all formulations of the problem.

Ringleb and Wiggins (1990) take an empirical approach to the problem. They study industries with high product liability and explore the entry of small companies into these industries with the advent of liability rules for worker hazards. They find that a statistically significant number of firms entered these industries with the advent of the new liability rules for workplace hazards. They posit that this entry is primarily through divestiture, in order to reduce liability overall to the larger firms from which the smaller firms were created. However, there are two caveats with their approach. One, they address worker liability and not hazardous waste disposal more generally. Two, it is unclear whether firms can avoid liability simply through divestiture. This paper offers a formal model to try to explain their empirically documented related phenomenon.

Finally, previous research by Boyd and Ingberman (1999, 2003) employ models similar to ours in many respects, but a few key differences should be noted. In their 1999 paper, the authors model a punitive damages situation where firms first choose a level of capital (that may later be exposed to liability) and then choose “deterrence.” They find that excess punitive damages will diminish the capital investment decision, and hence, the amount of care that firms take in response. Their model differs crucially from ours in two respects. First, they do not assume that the size of the loss depends on the level of care taken; it merely affects the probability of a loss. Our assumption is that the levels of care and loss are directly related. Second, they assume that the loss depends on the actions of only one actor. In contrast, we consider multiple tortfeasors and hence compare joint and several liability and several only liability. By examining two actors, and the associated behaviors of this type of situation, we can begin to build a more complete picture of the effects of legal rules on firms.

In their 2003 paper, Boyd and Ingberman model a situation where extended liability can induce firms to go bankrupt and disappear (or “fly by night”). They find that in some situations, extended

4See chapter 7, page 182.
liability can have unintended consequences by causing firms to exit the market, rather than incur liability. This happens because the costs of the liability exceed the benefits of staying in the market. Again, however, their model differs critically from ours. Specifically, they examine a producer and contractor situation. In their setup, there is no strategic interaction between the actors; rather, there is perfect monitoring of actions by each. As a result of this setup, they analyze situations where the producer and the contractor can cooperate via side payments around the legal rule. This is a very interesting situation and may be an interesting avenue for further research. However, as noted in the Introduction, this type of situation is less likely to be applied in situations of waste disposal. For this reason, we address the main issue, namely situations where firms cannot cooperate and may strategically interact in order to avoid liability.

3 An Example

We begin with a simple example to illustrate the effects of the choice of liability regime on the solvency and generation decisions of firms. There are two companies, 1 and 2. Each wants to build a plant in an industrial park with a shared waste management system. In order to do so, each must raise funds. Each firm must make two decisions. First, it must determine its scale of operation as determined by the amount of funds it raises. Second, it must allocate these funds to inputs.

The firm uses two types of assets to produce its output: “dirty” assets and “clean” assets. In the event of bankruptcy, dirty assets have no liquidation value because they have no use that does not generate liability. A contaminated hazardous waste site itself provides the clearest example of a dirty asset. Once contaminated, the site has little or no economic value. Clean assets, by contrast, do have liquidation value in bankruptcy because they may be used to produce goods to which no liability attaches. An example would be capital equipment that generates no waste and that is used to manufacture a good intermediate to the production of the final output of the firm.

Each firm first raises funds on the capital markets. It then allocates those funds between clean and dirty assets. With some abuse of terminology, we usually call dirty assets “waste”. This allocation depends on three factors: the production technology, the nature of the externality and the legal rule. The production technology determines the relative value of clean and dirty assets in production. The nature of the externality determines the scope of harm caused by the firms’ activities.

The governing legal rule dictates how the companies share responsibility for environmental damage. We compare two legal rules: joint and several liability and several only liability. They differ in one important respect. Under joint and several liability, if one company has insufficient assets to pay for its share of costs, these costs are borne by the other company (to the extent its solvency permits). Under several only liability, the unfunded share of costs of an insolvent company are not shifted to the other company. The probability of becoming insolvent depends on the amount invested in clean assets.

\[5\] In the formal model we use the level of waste produced as an index of the firm’s investment in dirty assets.
assets. In turn, the amount invested in clean assets depends on the legal rule. Thus, the probability of becoming insolvent is endogenous to the legal regime.

The production process, the nature of the externality and the legal rule imply that the decisions of one company affects the decisions of the other company. Thus company 1’s investment in clean assets and generated waste affect company 2’s investment in clean assets and generated waste. This implies that the probabilities of solvency are correlated through firm 1 and firm 2’s actions.

We show the following. For a given level of funds, company 1 and company 2 invest less in clean assets under joint and several liability than under several only liability. But company 1 and company 2 operate at a smaller scale – they raise less funds – under a rule of joint and several liability than under a rule of several only liability. Investment is more risky under joint and several liability than under several only liability because for a given level of funds and clean assets, joint and several liability increases the expected payments of a solvent firm relative to several only liability. Furthermore, there is no offsetting positive return under joint and several liability. Thus, the firms raise less funds under joint and several liability than under several only liability, and operate at a smaller scale. The smaller scale further reduces the level of assets available to creditors in the case of insolvency, and potentially, less waste is produced overall. The companies are more likely to become insolvent and less likely to pay their share of liability.

4 The Model

With this example in mind, we begin the description of our formal model.6

There are three types of agents in our model – investors, firms, and a regulator. In order to simplify the analysis, we assume that there are a continuum of perfectly competitive investors and two profit-maximizing firms, \( i = 1, 2 \). These agents take actions in four periods. A legal rule exists in all periods, and all agents know the legal rule. Figure 1 shows the timing of the game.

Figure 1: Timing of the Game

<table>
<thead>
<tr>
<th>Period 0</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investors choose ( W_i )</td>
<td>Firm ( i ) chooses ( x_i, k_i )</td>
<td>Firm may become insolvent</td>
<td>Costs allocated to any remaining solvent firms according to the prevailing legal rule.</td>
</tr>
</tbody>
</table>

4.1 The investment decision

In period 0, investors choose the amount of funds to invest in firms 1 and 2. Denote the total amount of funds firm \( i \) raises by \( W_i \). We assume that these funds are cash, and abstract from any issues

6Much of the notation in this paper parallels that of Kornhauser and Revesz (1989).
involving residual claimants to investors based on equity or debt issuance. Moreover, we assume that these investors have no control over the firm’s production decisions, other than providing funds. That is, once investors provide funds to firms, it is incumbent on firms to allocate these funds appropriately. Investors do know the technology that the firm uses, and have an expectation on the returns to this technology. Moreover, we assume that the investors know the legal rule that will determine the allocation of costs of cleanup in period 3.

4.2 The production process

In period 1, firms first allocate the funds generated between clean and dirty assets. They then produce. Both firms use the same production technology to produce identical goods. We denote firm $i$’s allocation to clean assets $k_i$, and its allocation to dirty assets $x_i$, which we call waste. All firms face the same prices for inputs. We normalize the price of $x_i$ to 1. Let the price of $k_i$ be $r_k$. Note that $r_k$ could potentially be interpreted as the return on assets in the market. We assume that the sum of $r_k k_i$ and $x_i$ must be less than or equal to $W_i$.

Production requires both clean assets and waste. The technology, however, exhibits decreasing returns to scale. We must further specify whether waste and clean assets are complements or substitutes in production. We assume that clean assets and waste are never perfect substitutes in production. In equilibrium, then, all firms allocate some financial capital to clean assets and some to waste.

Formally, we can express this as follows. First we decompose firm $i$’s profit function into a revenue or benefit function $B_i(x_i, k_i)$ and into a cost function $c_i(x_i, k_i)$. We assume that the benefit function is strictly concave, continuous, and three times differentiable, which implies

$$\frac{\partial B_i(x_i, k_i)}{\partial x_i} > 0, \frac{\partial B_i(x_i, k_i)}{\partial k_i} > 0, \frac{\partial^2 B_i(x_i, k_i)}{\partial x_i^2} < 0, \frac{\partial^2 B_i(x_i, k_i)}{\partial k_i^2} < 0,$$

and

$$\frac{\partial B_i(x_i, k_i)}{\partial x_i \partial k_i} > 0 \text{ or } \frac{\partial B_i(x_i, k_i)}{\partial x_i \partial k_i} < 0.$$

The last expression depends on whether the clean assets and generated waste are complements or substitutes.

Furthermore, we assume that

$$\lim_{x_i \to 0} \frac{\partial B_i}{\partial x_i} = \infty \text{ and } \lim_{x_i \to 0} \frac{\partial B_i}{\partial k_i} = \infty;$$

$$\lim_{x_i \to \infty} \frac{\partial B_i}{\partial x_i} = 0 \text{ and } \lim_{x_i \to \infty} \frac{\partial B_i}{\partial k_i} = 0.$$

\footnote{Our identification of dirty assets with the amount of waste generated slightly abuses terminology. An assumption that there is a one-to-one correspondence between the level of waste generated and the optimal allocations of clean and dirty assets justifies our terminological abuse.}
which imply that all firms use clean and dirty assets in equilibrium.

The costs of production depend on the level of clean and dirty assets deployed. For tractability, we assume separability in the cost function, and assume that \( k \) affects production costs while \( x \) affects liability costs. Thus we write the associated cost function, \( c(k_i) \) as a function only of the clean assets. This cost function is identical across firms, is strictly positive and increasing, continuous, and three times differentiable with \( c'(k_i) > 0 \) and \( c''(k_i) \geq 0 \). These costs are a function of the level of inputs.

The above assumptions imply firm \( i \)'s profit function,

\[
\Pi_i (x_i, k_i) = B_i (x_i, k_i) - c(k_i).
\]

The benefit of producing less the cost of production yields the firm’s profits. We assume that the firms operate in a perfectly competitive environment, thereby eliminating possible strategic effects that might occur if the two firms were operating in an oligopolistic market.8

The firm faces one more potential cost – damages for polluting the environment. Because we have assumed the same production technology for both firms, we may further assume that the waste generated by both firms is physically identical. Furthermore, the impact of waste generated on the environment is cumulative. While we assume that the marginal private benefits of generating waste decrease as the amount of waste increases, we assume that the social loss grows with the total amount of waste generated and that this loss increases at an increasing rate.

We define the loss function to be \( L = L(x_1, x_2) \). This function captures the cost to society caused by the waste of both firms. Assuming that firms emit physically identical waste, the loss function is equivalent to

\[
L(x_1, x_2) = L(x_1 + x_2) = L(X).
\]

We assume that this function is strictly convex, continuous, and three times differentiable, which implies that

\[
\frac{\partial L}{\partial x_1} = \frac{\partial L}{\partial x_2} > 0; \frac{\partial^2 L}{\partial x_i^2} = \frac{\partial^2 L}{\partial x_i \partial x_j} > 0; i, j = 1, 2; i \neq j.
\]

Furthermore, no waste created implies no loss to society, and an infinite amount of waste created implies an infinite societal cost, or

\[
\lim_{x_i \to 0} \frac{\partial L}{\partial x_i} = 0 \quad \text{and} \quad \lim_{x_i \to \infty} \frac{\partial L}{\partial x_i} = \infty.
\]

It is possible that the environmental loss function is discontinuous. For example, costs of cleanup may rise continuously with the volume of contaminated soil but jump discontinuously when ground water is contaminated. As no effective technology for cleansing the ground water exists (other than pump and use), the cost of cleanup effectively becomes infinite. Thus, included under the umbrella

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8See Daughety and Reinganum (2003).
of a convex loss function is an infinite cleanup cost; that is, after a certain level of waste, the loss to society would become infinite. In this case, no firm would be able to pay their apportioned liability.

4.3 Insolvency

We now address the problem of insolvency. In period 2, firms may become insolvent. Insolvency occurs after firms produce and receive benefits from production, that is, the flow of profits precedes the flow of liability. We assume that the probability that a firm becomes insolvent depends on the amount of clean assets used by the firm. We assume that as the amount of firm $i$’s clean assets increases, the expectation and the variance of the probability of insolvency decrease. Formulating solvency in terms of a probability of being able to pay all of the liability conceals all the technicalities of bankruptcy. However, using the probability formulation makes the problem tractable and allows us to focus the analysis on ex ante behavior effects.

4.4 The legal rule

In period 3, cleanup costs are allocated to any remaining solvent parties according to the prevailing legal rule. We assume that, between fully solvent parties, costs are allocated proportional to the amount of waste each firm generates. We assume that, if the firm is insolvent, it pays none of its share of the liability.

Our analysis compares two legal rules. The first is a rule of several only liability (SOL) under which a solvent firm is liable only for its apportioned share of the cost of waste deposited at a site. The second rule is joint and several liability with contribution (JSL), under which each firm is potentially held liable for more than its share of the waste deposited at a site. If both parties are solvent (meaning able to pay their liability) and if the costs of litigation are independent of the rule, then the two rules are equivalent. The two rules differ primarily in who bears the costs of an insolvent defendant – either society (SOL) or another defendant (JSL).

Thus, joint and several liability differs importantly from several only liability because, under joint and several liability, each party may be forced to bear the losses created by the other party. Each firm can reduce its expected liability in either (or both) of two ways: it can decrease the amount of waste it generates, thereby reducing the total amount and its share of liability, or it can reduce the amount of assets it has, thereby reducing the probability that it will pay any of the environmental costs.

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9 Furthermore, we assume a monotone hazard rate for the underlying distribution – two distributions that exhibit this property are the exponential and the normal.
10 The legal regime of joint and several liability is very complex, so that regimes of joint and several liability may differ on many dimensions. In the context of two tortfeasors, however, all regimes will have the property we assume: that each firm is responsible for its own share if both are solvent, and for the entire liability if the other tortfeasor is insolvent and unable to pay any of its share of liability.
11 Note that we are ignoring litigation costs in this framework. One legal rule may dominate the other in terms of social efficiency depending on the distribution of these costs.
Let $E[D^l_i]$ denote the expected liability of firm $i$ under regime $l$. Under several only liability, the expected liability of firm $i$ is

$$\text{Expected liability under SOL} = E[D^{SOL}_i] = (1 - P_i(k_i)) s_i L(X),$$

where $s_i \equiv x_i / (x_i + x_j)$, or firm $i$'s apportioned share of the waste.

Similarly, under joint and several liability, the expected liability is

$$\text{Expected liability under JSL} = E[D^{JSL}_i] = (1 - P_i(k_i)) (s_i + s_j P_j(k_j)) L(X).$$

These expressions imply that, regardless of the legal rule, the expected liability of firm $i$ depends critically on both the assets and waste generation of the firm $j$. Under several only liability, the expected liability of firm $i$ depends explicitly on the waste generation of firm $j$ and implicitly on the assets of firm $j$, while under joint and several liability, the expected liability of firm $i$ depends explicitly on both the waste generation and assets of firm $j$.

The question remains as to how the expected liability changes with an increase in clean assets. First, assume that the allocation constraint is binding, or $r_k k_i + x_i = W_i$. Holding the decisions of firm $j$ constant, under several only liability, we have, for a change in $k_i$ the following change in expected liability under several only liability:

$$\frac{\partial E[D^{SOL}_i]}{\partial k_i} = - \frac{\partial P_i}{\partial k_i} s_i L(X) - r_k (1 - P_i) \left( \frac{1}{X} (1 - s_i) L(X) + s_i \frac{\partial L}{\partial x_i} \right).$$

There are two groups of effects of the increase in $k_i$ under several only liability. First, an increase in $k_i$ causes an increase in the probability of solvency when cleanup costs are realized, thereby increasing expected liability. Second, an increase in financial capital allocated to $k_i$ necessarily decreases the financial capital allocate to waste production. The decrease has an effect on (1) the distribution of the shares of waste for which the tortfeasors will be held liable and (2) the magnitude of the environmental loss. Holding the decisions of the other firm constant, a decrease in the level of waste generated by firm $i$ will cause a narrowing of the distribution of apportioned shares and a decrease in the magnitude of environmental loss. Equilibrium depends on which effect dominates.

Under joint and several liability, we have the following:

$$\frac{\partial E[D^{JSL}_i]}{\partial k_i} = - \frac{\partial P_i}{\partial k_i} (s_i + s_j P_j) L(X) - r_k (1 - P_i) \left( - \frac{1}{X} s_j P_j L(X) + s_j P_j \frac{\partial L}{\partial x_i} \right).$$
In order to understand the key difference between several only liability and joint and several liability, note that we can rewrite \( \frac{\partial E[D_{iSL}^j]}{\partial k_i} \) in terms of \( \frac{\partial E[D_{iSOL}^j]}{\partial k_i} \), i.e.

\[
\frac{\partial E[D_{iSL}^j]}{\partial k_i} = \frac{\partial E[D_{iSOL}^j]}{\partial k_i} + s_jP_j \left[ -\frac{\partial P_i}{\partial k_i} L(X) - r_k (1 - P_i) \left( -\frac{1}{X} L(X) + \frac{\partial L}{\partial x_i} \right) \right].
\]

There are three groups of effects of the increase in \( k_i \) under joint and several liability. The first two are identical to those under several only liability – the direct solvency effect and the indirect distribution and magnitude environmental loss effects. The third effect comes from the interaction between firms, and specifically, the increase in firm \( i \)'s expected liability for firm \( j \)'s share. The first term in brackets represents the direct effect of an increase in \( k_i \) on the probability of paying firm \( j \)'s apportioned share. The second term is an indirect effect of an increase in \( k_i \). An increase in \( k_i \) causes a decrease in firm \( i \)'s share, which implies an increase in firm \( j \)'s share. An increase in \( k_i \) also causes a decrease in the environmental loss. But, because \( L(\cdot) \) is strictly convex, \( \left( -\frac{1}{X} L(X) + \frac{\partial L}{\partial x_i} \right) \) is negative. Firm \( j \)'s increase in liability due to share redistribution is larger than firm \( j \)'s decrease in expected liability due to incrementally less environmental damage. Hence, the expected liability under JSL is the expected liability under SOL, plus a positive factor that accounts for the increase in potential liability due to firm \( j \).

The firms choose their input allocations noncooperatively. Each firm’s choice of \( k_i \), as virtue of the reaction functions, necessarily depends on the choice of \( k_j \). In this sense, the probabilities of insolvency are correlated.

Our formulation of the problem of insolvency implies that a solvent firm pays the full amount of the bankrupt’s share of liability. This formulation assumes that the bankrupt has no assets available to distribute to the creditor who otherwise bears the cleanup costs. This outcome is most likely if cleanup costs have the lowest priority. The actual law is unclear on the issue of priority.

5 Benchmark: Maximizing social welfare

Our exposition from this point forward examines the period 1 firm decisions first, treating the funds \( W_i \) as given. Then, we investigate the period 0 investor decision. We use the social welfare maximizing solution as a benchmark.

To determine the social welfare maximizing solution, we assume that there exists a social planner who chooses the amounts of inputs employed by both firms. The social planner will choose to maximize the sum of the benefits to the two firms of using both types of inputs, less the cost to society of waste production.\(^{12}\) Thus, our model does not assume that social welfare is maximized in a “no-pollution” environment. This is realistic, as almost all processes create some environmental

\(^{12}\)The goal of a legal liability rule may or may not be the achievement of these particular optimal conditions, and the rule may add or subtract other components to society’s welfare.
The social planner chooses the input allocation for both firms simultaneously, which internalizes all externalities that from the firms’ shared cleanup responsibility. The social planner also assumes all firms are perfectly solvent when liability is assessed, or equivalently, places the burden for cleanup squarely on the two firms. These assumptions characterize the social optimum, defined as the solution to this problem.

In terms of our model, the social planner allocates funds to solve

\[ \Gamma^* = \max_{x_1, k_1, x_2, k_2} B_1(x_1, k_1) + B_2(x_2, k_2) - c(k_1) - c(k_2) - L(X) \]  

subject to the constraints

\[ r_k k_1 + x_1 \leq W_1^* \quad \text{and} \quad r_k k_2 + x_2 \leq W_2^*. \]  

In this sense, the social planner takes the value of the firms given by the market, and chooses the optimal input allocation, subject to these constraints. If we let the technologies of the two firms differ, the social planner may decide to allocate more funds to the more environmentally efficient firm in order to maximize social welfare. For comparison purposes, we keep the technologies the same, and let investors determine the optimal level of funds to invest.

Let \( \lambda_i, i = 1, 2 \) denote the Lagrangean multipliers associated with the constraints in (2). The first order conditions for this problem are

\[ \frac{\partial \Gamma^*}{\partial x_i} = \frac{\partial B_i}{\partial x_i} - \frac{\partial L}{\partial x_i} - \lambda_i \leq 0, \quad x_i \geq 0, \quad x_i \frac{\partial \Gamma^*}{\partial x_i} = 0, \]  

\[ \frac{\partial \Gamma^*}{\partial k_i} = \frac{\partial B_i}{\partial k_i} - c'(k_i) - \lambda_i r_k \leq 0, \quad k_i \geq 0, \quad k_i \frac{\partial \Gamma^*}{\partial k_i} = 0, \quad \text{and} \quad i = 1, 2. \]  

The social planner sets the marginal benefit of generating waste equal to the marginal cost of generating waste, and likewise for the marginal benefit of creating clean assets equal to the marginal cost of creating clean assets.

Assuming an interior solution implies that the constraints are binding. Solving the first order conditions gives the socially optimal levels of \( x^*_i(W_i, W_j, r_k) \) and \( k^*_i(W_i, W_j, r_k) \), \( i = 1, 2, \) \( i \neq j \) as a function of the funds raised by each firm and the price of clean inputs.

Substituting the socially optimal allocations yields our baseline level of social welfare,

\[ \Lambda^* = B_1(x_1^*, k_1^*) + B_2(x_2^*, k_2^*) - c(k_1^*) - c(k_2^*) - L(x_1^* + x_2^*). \]
We can interpret $\Lambda^*$ as the amount of funds a social planner would allocate to the project without any possibility of insolvency, or, the welfare maximizing certainty equivalent.

- **Comparative statics**

As a benchmark, we note that for a given level of $W_j$, the amount of an increase in the funds raised by firm $i$ causes both waste and clean assets to increase for firm $i$, and waste to decrease and clean assets to increase for firm $j$. The intuition behind this result is as follows. An increase in the funds raised by firm $i$ necessarily implies an increase in waste produced by firm $i$. Thus the social planner chooses to allocate less funds to waste for firm $j$ in order to keep the total marginal benefit of waste production equal to the total marginal cost.

This analysis leads to our first lemma,

**Lemma 1** *For a given level of $W_j$, $\frac{dk_i}{dW_i} > 0$*

### 6 The Firms’ Problem

With the above benchmark in mind, we now examine the optimal actions of firms. As a first step, we examine a situation with no liability. This situation documents the incentives of firms to produce waste in a situation without environmental regulation – similar to pre-CERCLA situations. The next step is to examine the effects of a legal rule. We examine two different scenarios – several only liability and joint and several liability. Our major goal is to examine the externality created by the legal rule on input allocation. The analysis shows that the legal rule critically affects input decisions differently under the two legal regimes. Moreover, the incentive for insolvency differs under the two legal regimes.

#### 6.1 No liability

In a situation with no liability, the firms raise funds in period 0 and make input allocation decisions in period 1. The firms act simultaneously and non-cooperatively in choosing how to allocate funds between inputs that create waste and inputs that create clean assets. Period 2, where firms can become insolvent, and period 3, where liability is assessed, are not addressed. We solve the period 1 problem first, taking funds $W_i$ as given, and then investigate the optimal levels of $W_i$.

- **The maximization problem**

Under no only liability, in period 1, firm $i$ allocates funds $W_i$ to solve

$$
\Gamma_i = \max_{x_i, k_i} B_i (x_i, k_i) - c(k_i)
$$

(7)
subject to the constraint
\[ r_k k_i + x_i \leq W_i, \quad i = 1, 2. \] (8)

The first order conditions for this problem are
\[
\frac{\partial \Gamma_i}{\partial x_i} = \frac{\partial B_i}{\partial x_i} - \lambda_i \leq 0, \quad x_i \geq 0, \quad x_i \frac{\partial \Gamma_i}{\partial x_i} = 0,
\]
\[
\frac{\partial \Gamma_i}{\partial k_i} = \frac{\partial B_i}{\partial k_i} - c' (k_i) - \lambda_i r_k \leq 0, \quad k_i \geq 0, k_i \frac{\partial \Gamma_i}{\partial k_i} = 0, \quad \text{and}
\]
\[
\frac{\partial \Gamma_i}{\partial \lambda_i} = W_i - r_k k_i - x_i \geq 0, \quad \lambda_i \geq 0, \quad \lambda_i \frac{\partial \Gamma_i}{\partial \lambda_i} = 0, \quad i = 1, 2.
\]

Assuming an interior solution implies that the constraints are binding. Solving the first order conditions gives the no liability levels of \( x_{i\text{NL}} (W_i, W_j, r_k) \) and \( k_{i\text{NL}} (W_i, W_j, r_k), \quad i = 1, 2, \ i \neq j \) as a function of the funds raised by each firm and the price of clean inputs.

### 6.2 Several only liability

Our two firms are now simultaneously and non-cooperatively choosing how to allocate funds between inputs that create waste and inputs that create clean assets. According to the timing of the game, we assume that firms raise funds in period 0, and make input allocation decisions in period 1. In period 2, firms 1 and 2 potentially become insolvent, and in period 3, liability is assessed. Again, we solve the period 1 problem first, taking funds \( W_i \) as given, and then investigate the optimal levels of \( W_i \).

- The maximization problem

Under several only liability, in period 1, firm \( i \) allocates funds \( W_i^{\text{SOL}} \) to solve
\[
\Gamma_i^{\text{SOL}} = \max_{x_i, k_i} B_i (x_i, k_i) - c (k_i) - (1 - P_i) s_i L (X)
\]
subject to the constraint
\[ r_k k_i + x_i \leq W_i^{\text{SOL}}, \quad i = 1, 2. \] (12)

The first order conditions for this problem are
\[
\frac{\partial \Gamma_i^{\text{SOL}}}{\partial x_i} = \frac{\partial B_i}{\partial x_i} - (1 - P_i) \left( \frac{1}{X} s_j L (X) + s_i \frac{\partial L}{\partial x_i} \right) - \lambda_i \leq 0, \quad x_i \geq 0, \quad x_i \frac{\partial \Gamma_i^{\text{SOL}}}{\partial x_i} = 0,
\]
\[
\frac{\partial \Gamma_i^{\text{SOL}}}{\partial k_i} = \frac{\partial B_i}{\partial k_i} - c' (k_i) + \frac{\partial P_i}{\partial k_i} s_i L (X) - \lambda_i r_k \leq 0, \quad k_i \geq 0, k_i \frac{\partial \Gamma_i^{\text{SOL}}}{\partial k_i} = 0, \quad \text{and}
\]
\[
\frac{\partial \Gamma_i^{\text{SOL}}}{\partial \lambda_i} = W_i^{\text{SOL}} - r_k k_i - x_i \geq 0, \quad \lambda_i \geq 0, \quad \lambda_i \frac{\partial \Gamma_i^{\text{SOL}}}{\partial \lambda_i} = 0, \quad i = 1, 2.
\]
Assuming an interior solution implies that the constraints are binding. Solving the first order conditions gives the optimum as a function of the funds raised by each firm in the market, the price of assets, and the time of the realization of cleanup costs, namely \( x_i^{SOL}(W_i^{SOL}, W_j^{SOL}, r_k) \) and \( k_i^{SOL}(W_i^{SOL}, W_j^{SOL}, r_k), i = 1, 2, i \neq j. \)

Substituting these values into the welfare equation yields the level of welfare under several only liability:

\[
\Lambda^{SOL} = B_1 \left( x_1^{SOL}, k_1^{SOL} \right) + B_2 \left( x_2^{SOL}, k_2^{SOL} \right) - c \left( k_1^{SOL} \right) - c \left( k_2^{SOL} \right) - L \left( x_1^{SOL} + x_2^{SOL} \right)
\]

Investigating the equilibrium

As a first step, we want to look at how a hypothetical change in the level of inputs allocated to waste production from firm 1 will affect the input decisions of firm 2, holding \( W_1 \) and \( W_2 \) constant. Note we are not describing the ultimate equilibrium; rather, we are just discussing how firm 2 may react to a change in firm 1’s allocation decision.

Suppose that firm 1 increases the share of inputs allocated to waste production. Assuming that firm 1 allocates all funds to one of the two inputs, this necessarily implies that firm 1 decreases its use of the clean input. Firm 1’s action has two effects. First, the total amount of environmental waste increases. Second, the probability that firm 1 is solvent decreases. Both cause firm 2’s expected costs to increase – the first through the increased environmental waste, the second through the decreased solvency.

There are two possible actions for firm 2. The first possible action is that firm 2 can choose to increase its share of inputs allocated to waste production. This necessarily implies that firm 2 decreases the share of inputs allocated to clean asset production. Again, this has two effects. Firm 2’s expected costs increase as a result of increased environmental waste, but decrease due to decreased solvency.

The second possible action is that firm 2 can choose to decrease waste production. This necessarily implies that firm 2 increases its production of clean assets. Firm 2’s expected costs decrease due to decreased waste, but increase from increased solvency.\(^{13}\)

Thus, the actions of firm 1 affect the actions of firm 2. This implies that firm 2’s probability of solvency depends on firm 1’s probability of solvency. In this way, the probability that firm 2 is solvent is correlated with the probability that firm 1 is solvent. Whether this correlation is positive or negative depends on our assumptions regarding the optimal reactions of firms.

The discussion above points to multiple equilibria for this problem. For the purposes of this paper, it is more instructive to examine the region where the equilibria change as a result of the legal

\(^{13}\)Everything also works in reverse. If firm 1 increases its use of the clean input, it necessarily decreases its use of the dirty input. Both cause firm 2’s expected costs to decrease. Firm 2 may decide to increase its use of the dirty input, or firm 2 may decide to increase its use of the clean input.
rule, and where the constraints on assets and waste are binding. This rules out equilibria where one or both of the firms exit (or never enter) production, because the expected cost of cleanup is too high.

We assume that we are examining a scenario where the reaction function of firm $i$ is decreasing in $x_j$, or equivalently,

$$x'_i(x_j) < 0,$$

where $x'_i(x_j)$ denotes the reaction function of firm $i$ to a change in the amount of dirty input used by firm $j$.

The Appendix derives the reaction functions. For the equilibrium to be stable, we must have:

$$|x'_i(x_j)||x'_j(x_i)| < 1. \quad (16)$$

and we can now show the following Proposition.

**Proposition 2** For given levels of $W_i$ and $W_j$, $x^{\text{SOL}}_i > x^*_i$ and $k^{\text{SOL}}_i < k_i^*$.

**Proof.** See the Appendix. □

Thus we find that firms generate more waste and go insolvent more often under several only liability than would be socially optimal. Firms do not internalize their entire cost of waste production; hence, we see increased levels of waste.

### 6.3 Joint and several liability

Under several only liability, we know that firms 1 and 2 will be liable for, at most, its respective apportioned share of the total liability. However, under joint and several liability, there is a distinct possibility that given the amount of clean assets already chosen by the two firms, if firm 1 becomes insolvent, firm 2 will have to pay the liability of firm 1, and vice versa. This possibility resulting from a rule of joint and several liability creates an incentive for the firms to expand their generation of waste, for any given scale of investment by the two firms, beyond the levels chosen under several only liability.

- **The maximization problem**

Under joint and several liability, firm $i$ chooses to allocate funds $W_i$ to solve

$$\Gamma_i^{JSL} = \max_{x_i, k_i} B_i(x_i, k_i) - c(k_i) - (1 - P_i) (s_i + s_j P_j) L(X) \quad (17)$$

subject to the constraint

$$r_k k_i + x_i \leq W_i, \ i, j = 1, 2, i \neq j. \quad (18)$$
The first order conditions for this problem are

\[
\frac{\partial \Gamma_{JSL}}{\partial x_i} = \frac{\partial B_i}{\partial x_i} - (1 - P_i) \left( \frac{1}{X} s_j L(X) + s_i \frac{\partial L}{\partial x_i} \right) - \lambda_i \leq 0, \quad x_i \geq 0, \quad x_i \frac{\partial \Gamma_{JSL}}{\partial x_i} = 0, \quad (19)
\]

\[
\frac{\partial \Gamma_{JSL}}{\partial k_i} = \frac{\partial B_i}{\partial k_i} - c' (k_i) + \frac{\partial P_i}{\partial k_i} (s_i + s_j P_j L(X) - \lambda_i r_k \leq 0, \quad k_i \geq 0, \quad k_i \frac{\partial \Gamma_{JSL}}{\partial k_i} = 0, \quad (20)
\]

\[
\frac{\partial \Gamma_{JSL}}{\partial \lambda_i} = W_{JSL}^i - r_k k_i - x_i \geq 0, \quad \lambda_i \geq 0, \quad \lambda_i \frac{\partial \Gamma_{JSL}}{\partial \lambda_i} = 0, \quad i = 1, 2.
\]

Assuming an interior solution will imply that the constraints are binding. Solving the first order conditions gives the optimum as a function of the funds raised by each firm in the market, the price of assets, and the time of the realization of cleanup costs, namely \( x_{iJSL} (W_i, W_j, r_k) \) and \( k_{iJSL} (W_i, W_j, r_k) \), \( i = 1, 2, \ i \neq j \).

Substituting these values into the welfare equation yields the level of welfare under joint and several liability:

\[
\Lambda_{JSL} = B_1 (x_{1JSL}^i, k_{1JSL}^i) + B_2 (x_{2JSL}^i, k_{2JSL}^i) - c (k_{1JSL}^i) - c (k_{2JSL}^i) - L (x_{1JSL}^i + x_{2JSL}^i). \quad (21)
\]

• Investigating the equilibrium

Just as with several only liability, our problem is subject to multiple equilibria. Again, however, we assume that we are examining a scenario where the reaction function of firm \( i \) is decreasing in \( x_j \), or equivalently, \( x_i' (x_j) < 0 \). At the same time, this implies that \( x_i' (k_j) > 0 \).

Similar to several only liability, we have the following Proposition.

**Proposition 3** For given levels of \( W_i \) and \( W_j \), \( x_{iJSL}^i > x_i^* \) and \( k_{iJSL}^i < k_i^* \).

**Proof.** See the Appendix. ■

Again, we find that firms generate more waste and go insolvent more often than would otherwise be the social optimum. Firms do not internalize their entire cost of waste production; hence, we see increased levels of use of dirty inputs.

**7 Comparison of equilibria**

We know that because the firms are acting non-cooperatively, under both legal regimes, the amount of waste each generates in equilibrium will be greater than the socially optimal amount of waste, and firms go bankrupt more often than in the social optimum. We now ask how the two equilibria compare to each other.

• Several only liability versus joint and several liability
We can now show the following:

**Proposition 4** For given levels of $W_i$ and $W_j$, $x_i^{JSL} > x_i^{SOL}$ and $k_i^{JSL} < k_i^{SOL}$.

**Proof.** See the Appendix. □

Thus we find, at an interior optimum, that firms go bankrupt more often under joint and several liability than under several only liability, and generate more waste under joint and several liability than under several only liability, for a given level of funds.

The key to understanding these results are the assumptions made on the expected liability in the case of insolvency. If we assume that the expected value of the liability of a firm increases in assets, costs of clean assets increase faster under joint and several liability than under several only liability. This implies that the equilibrium amount of clean assets is lower under joint and several liability than under several only liability.

This claim is intuitively appealing – as an intermediate result. As its investment in clean assets increases, joint and several liability always implies a higher expected liability than several only liability for a given scale of total investment. Firms recognize this, and adjust input allocation to reduce this expected liability.

• The investor’s decision

All of these conclusions so far have been made under fairly restrictive assumptions. But one thought experiment will provide an intuition for our final result that the scale of total investment by firms under joint and several liability is less than the scale under several only liability, implying that firms go insolvent more often under joint and several liability, but potentially create less total waste.

Consider a risk averse investor deciding how to invest funds, $W_0$. We know that if the asset has a positive return, the investor will place at least some funds in the risky asset. Let $W_i$ be the amount of funds that the investor puts into a firm that is subject to liability for waste generation. We have seen that firms will adjust their waste generation and the level of clean assets in response to the liability rule. If we assume that these firms operate in a competitive marketplace, and if firms are able to raise the same amount of funds under both liability regimes, $W_i$, then the expected return to firms under both liability rules should be equal. However, the variance of the investment under joint and several liability must be greater than the variance of the investment under several only liability. This implies that the risk averse investor would require a higher mean return on the investment under joint and several liability than under several only liability. Because there is no “upside” to this risk, due to our assumption that the expected liability increases in assets, this implies that the risk averse investor will invest fewer funds in a venture with higher variance than in one with

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14Under our assumption of the expected payout increasing in the level of assets, this must necessarily be the case. Another way of looking at this is that several only liability second order stochastically dominates joint and several liability.
lower variance. This implies that in equilibrium, risk averse investors will place fewer funds into companies that operate under joint and several liability than under several only liability.

**Proposition 5** $W_i^{JSL} \leq W_i^{SOL}$.

**Proof.** In order to show that this is the case, it is sufficient to show that several only liability second order stochastically dominates joint and several liability. We examine the expected profit functions of the firms,

$$
\Pi_i^{SOL} = B_i \left( x_i^{SOL}, k_i^{SOL} \right) - c \left( k_i^{SOL} \right) - \left( 1 - P_i^{SOL} \right) s_i^{SOL} L \left( X^{SOL} \right) \quad \text{and}
$$

$$
\Pi_i^{JSL} = B_i \left( x_i^{JSL}, k_i^{JSL} \right) - c \left( k_i^{JSL} \right) - \left( 1 - P_i^{JSL} \right) \left( s_i^{JSL} + s_j P_j^{JSL} \right) L \left( X^{JSL} \right).
$$

Assume that $\Pi_i^{SOL} = \Pi_i^{JSL}$. $\Pi_i^{JSL}$ puts a relatively greater share of the funds in the risky assets than $\Pi_i^{SOL}$. Assuming there is a riskless outside option in which to invest, these risk averse investors will put relatively more funds in the riskless asset under JSL than under SOL. This implies that there is less investment overall in firms under joint and several liability than under several only liability.

This decrease in funds under JSL implies that the firms are smaller. Smaller firms implies less waste, and less solvent firms.

One limitation is that we hold the number of firms constant, and the identities of these firms constant. If the number of firms is allowed to vary, and there are a greater number of smaller firms, total waste generation potentially increases. But, if we assume contestable markets, both several only and joint and several liability will produce a socially efficient number of firms. If costs are higher under joint and several liability than under several only liability, fewer firms, or smaller firms, may exist under joint and several liability than under several only liability.

**8 Conclusions and Suggestions for Further Research**

This paper extends the model in Kornhauser and Revesz (1990) by endogenizing the solvency decision of firms in the face of possible liability. We have found that an endogenous solvency framework causes firms to “disappear” more often under joint and several liability than under several only liability. Joint and several liability, however, may also reduce the amount of waste generated by decreasing the amount invested in these potentially waste generating firms. Although the government and taxpayers potentially bear a greater share of the cleanup cost under joint and several liability, it may be the case that the cleanup cost is lower under joint and several liability than under several only liability, due to lower amounts of waste produced. The change in investment in these firms and the resulting inefficiencies should not be ignored, however.

These results provide a different perspective on the comparison between joint and several liability than those of Kornhauser and Revesz [1990]. That model measures the scale of firms in terms of
solvency. When firms are large, joint and several liability induces firms to generate less waste than a rule of several only liability. When firms are very small by contrast, the two rules are equivalent. For "intermediate" sizes, joint and several liability induces firms to generate more waste than they would generate under several only liability because joint and several liability induces these intermediate sized firms to go insolvent while several only liability does not.

Our measure of firm scale differs; we measure scale by the amount of capital raised in capital markets. Under our model, when we fix a scale, joint and several liability induces the firm to generate more waste than it would under several only liability. Phrased differently, under joint and several only liability the firm allocates less capital to solvency and more to waste than it would under several only liability. When we make solvency endogenous, then we see that Kornhauser and Revesz thus implicitly compare a firm of fixed solvency $k$ and total capital $W$ under several only liability to a firm with larger capital $W' > W$ but identical capital $k$ under joint and several liability.

Our conclusions do not directly lead to policy prescriptions. Our results are derived under some special assumptions; policymakers require more confidence that our results are robust to changes in assumptions. In particular, we have assumed a fixed number of firms in the industry and depositing at a given site. Allowing entry into the industry or to the site may change our results.

There are many possible extensions of the model. One would be to relax the (implicit) assumption of perfect competition of the two firms in their primary production market. Strategic waste dumping may occur if the two firms compete in the market for goods or the market for funds, and are able to affect the equilibrium price in one or both of these markets. Furthermore, we do not address the case where firms may cooperatively decide allocations of waste and assets. Firms may have an incentive to do this if they operate in similar product markets, and can use environmental liability as a discipline device for price collusion in oligopoly markets. This presents an interesting avenue for further research. Another extension would be to examine the optimal number of firms dumping at a site. Other extensions include a full set of welfare calculations based on the various scenarios, and a general equilibrium approach in which the cost of funds varies with the legal regime. In a general equilibrium model, the cost of funds potentially should be higher under joint and several liability than under several only liability. This intuition strengthens the scale result presented in this paper.

The essence of the matter is this. Generating more waste increases the harm to the environment, and possibly increases liability. Higher assets raises exposure to liability. Under joint and several liability, the total amount of waste generated may decrease, but firms are more likely to go bankrupt than under several only liability. By explicitly incorporating solvency as a decision for the firm, we gain insight into the optimal behavior of firms under different liability rules.
References


A Mathematical appendix

This section offers proofs for claims made in the text.

A.1 Several only liability

- Investigating the equilibrium: \( x'_i(x_j) \)

We know that

\[
x'_i(x_j) = -\frac{\frac{\partial^2 \Gamma_{SOL}}{\partial x_i \partial x_j}}{\frac{\partial \Gamma_{SOL}}{\partial x_i^2}}
\]

Equilibrium implies that the denominator is negative. Thus the sign of the reaction function depends on the sign on the numerator.

\[
x'_i(x_j) = -\left( -\frac{\partial P_i}{\partial k_i} \right) \left( -\frac{1}{r_k} \right) s_i \left[ -\frac{L(X)}{X} + \frac{\partial L}{\partial x_j} \right] - \frac{(1-P_i)}{X} \left[ s_i \frac{L(X)}{X} + (1-s_i) \left[ -\frac{L(X)}{X} + \frac{\partial L}{\partial x_j} \right] \right]
\]

We first examine the numerator terms. The first term is negative, the second term is ambiguous and the third term is positive. The question remains as to which effect dominates. We can attempt to
answer this by looking at the denominator. The first line is negative, the second line is ambiguous, the third line is positive, and the fourth line is ambiguous. However, the denominator must be negative for equilibrium to hold. Working through these terms shows that a stable equilibrium implies that the numerator is negative.

**Proposition 1** For given levels of $W_i$ and $W_j$, $x_i^{SOL} > x_i^*$ and $k_i^{SOL} < k_i^*$.

**Proof.** This result is easier to see if we assume the constraints are binding, solve for the optimal decision under both regimes, and prove by contradiction.

We can rewrite (3) and (13) as

\[
\frac{\partial \Gamma^*}{\partial x_i} = \frac{\partial B_i}{\partial x_i} - \frac{\partial L}{\partial x_i} - \frac{\partial B_i}{\partial k_i} + r_k c'(k_i) = 0 \text{ and}
\]

\[
\frac{\partial \Gamma^*}{\partial x_i} = \frac{\partial B_i}{\partial x_i} - \left(1 - P_i\right) \left(\frac{1}{X} s_j L(X) + s_i \frac{\partial L}{\partial x_i}\right)
- \frac{\partial B_i}{\partial k_i} + r_k c'(k_i) - \frac{\partial P_i}{\partial k_i} s_i L(X)
= 0.
\]

Let $\frac{\partial B_i}{\partial x_i}$ denote the function $\frac{\partial B_i}{\partial x_i}$ evaluated at the point $(x_i^*, k_i^*)$, and likewise for $\frac{\partial B_i}{\partial x_i}$. In addition, let $X^* \equiv x_i^* + x_j(x_i^*)$, $X^{SOL} \equiv x_i^{SOL} + x_j(x_i^{SOL})$, $s_i^* \equiv x_i^*/(x_i^* + x_j(x_i^*))$ and $s_i^{SOL} \equiv x_i^{SOL}/(x_i^{SOL} + x_j(x_i^{SOL}))$. In order to show that $x_i^{SOL} > x_i^*$, we suppose not, that is, $x_i^{SOL} \leq x_i^*$. This leads to the following string of inequalities:

\[
0 = \frac{\partial B_i^*}{\partial x_i^*} - \frac{\partial L^*}{\partial x_i^*} - \frac{\partial B_i^*}{\partial k_i^*} + r_k c'(k_i^*)
< \frac{\partial B_i^*}{\partial x_i^*} - \frac{\partial B_i^*}{\partial k_i^*} + r_k c'(k_i^*) - \left(1 - P_i^*\right) \left(\frac{1}{X} s_j^* L(X^*) + s_i^* \frac{\partial L}{\partial x_i^*}\right)
- \frac{\partial P_i}{\partial k_i} s_i^* L(X^*)
\leq \frac{\partial B_i^{SOL}}{\partial x_i^{SOL}} - \frac{\partial L^{SOL}}{\partial x_i^{SOL}} - \frac{\partial B_i^{SOL}}{\partial k_i^{SOL}} + r_k c'(k_i^{SOL}) - \left(1 - P_i^{SOL}\right) \left(\frac{1}{X^{SOL}} s_j^{SOL} L(X^{SOL}) + s_i^{SOL} \frac{\partial L}{\partial x_i^{SOL}}\right)
- \frac{\partial P_i}{\partial k_i^{SOL}} s_i^{SOL} L(X^{SOL})
= 0.
\]

Contradiction.  ■

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A.2 Joint and several liability

- Investigating the equilibrium: $x'_i(x_j)$

The reaction function under joint and several liability is slightly more complicated than under several only liability, but all several only terms are included in the joint and several terms. There are also multiple equilibria for this problem, but again, for stability, the denominator must be negative.

The reaction function under joint and several liability is

$$x'_i(x_j) = \frac{-\left(-\frac{\partial B_i}{\partial x_i}\right)\left(-\frac{1}{r_k}\right) \left[\left(-s_i \frac{1}{X} (1 - P_j) + s_j \frac{\partial P_j}{\partial k_j} \left(-\frac{1}{r_k}\right)\right) L(X) + (s_i + s_j P_j) \frac{\partial L}{\partial x_i}\right]}{-\left(-\frac{\partial B_i}{\partial x_i}\right)\left(-\frac{1}{r_k}\right)^2 - \left(-\frac{\partial P_i}{\partial k_i}\right)^2 - \left(-\frac{\partial P_i}{\partial k_i}\right) \left(-\frac{1}{r_k}\right)^2 (s_i + s_j P_j) L(X) - 2 \left(-\frac{\partial P_i}{\partial k_i}\right) \left(-\frac{1}{r_k}\right) \left[\frac{L(X)}{X} + (s_i + s_j P_j) \frac{\partial L}{\partial x_i}\right] - (1 - P_i) \left(1 - s_i\right) \left[-\frac{1}{X} \frac{\partial L}{\partial x_i} + (s_i + s_j P_j) \frac{\partial^2 L}{\partial x_i^2}\right]}.$$  

**Proposition 2** For given levels of $W_i$ and $W_j$, $x_i^{JS} > x_i^*$ and $k_i^{JS} < k_i^*$.

**Proof.** Again, this result is easier to see if we assume the constraints are binding, solve for the optimal decision under both regimes, and prove by contradiction.

We can rewrite (3) and (19) as

$$\frac{\partial \Gamma}{\partial x_i} = \frac{\partial B_i}{\partial x_i} - \frac{\partial L}{\partial x_i} - \frac{\partial B_i}{\partial k_i} + r_k c' (k_i) = 0 \text{ and}$$

$$\frac{\partial \Gamma_i^{JS}}{\partial x_i} = \frac{\partial B_i}{\partial x_i} - (1 - P_i) \left\{ \frac{1}{X} s_j L(X) + s_j \frac{\partial L}{\partial x_i} \right\} - \frac{\partial B_i}{\partial k_i} + r_k c' (k_i) - \frac{\partial P_i}{\partial k_i} (s_i + s_j P_j) L(X) = 0.$$  

Let $\frac{\partial B_i^{JS}}{\partial x_i}$ denote the function evaluated at the point $(x_i^{JS}, k_i^{JS})$. In addition, let $X^{JS} = x_i^{JS} + x_j (x_i^{JS})$ and $s_i^{JS} = x_i^{JS} / (x_i^{JS} + x_j (x_i^{JS}))$. In order to show that $x_i^{JS} > x_i^*$, we suppose not, that is, $x_i^{JS} \leq x_i^*$. This leads to the following string of inequalities:

\[\text{\textbf{24}}\]
\[ 0 = \frac{\partial B^*_i}{\partial x^*_i} - \frac{\partial L^*_i}{\partial x^*_i} - \frac{\partial B^*_i}{\partial k^*_i} + r_k c'(k^*_i) \]

\[ < \frac{\partial B^*_i}{\partial x^*_i} - \frac{\partial B^*_i}{\partial k^*_i} + r_k c'(k^*_i) - (1 - P_i^*) \left( \frac{1}{X^*} s_j^* L(X^*) + s_i^* \frac{\partial L}{\partial x_i} \right) \]

\[ - \frac{\partial P_i}{\partial k^*_i} \left( s_i + s_j P_j^* \right) L(X^*) \]

\[ \leq \frac{\partial B^*_{iJSL}}{\partial x^*_i} - \frac{\partial B^*_{iJSL}}{\partial k^*_i} + r_k c'(k^*_{iJSL}) - \left( \frac{1}{X} s_j^{*JSL} L(X^{*JSL}) + s_i^{*JSL} \frac{\partial L}{\partial x_i^{*JSL}} \right) \]

\[ - \frac{\partial P_i}{\partial k^*_{iJSL}} \left( s_i^* + s_j P_j^{*JSL} \right) L(X^{*JSL}) \]

\[ = 0. \]

Contradiction.  ■

A.3 Comparison of equilibria

Proposition 5 For given levels of \( W_i \) and \( W_j \), \( x^*_i > x^*_i^{SOL} \) and \( k^*_{iJSL} < k^*_{iSOL} \).

Proof. Using the same logic of Propositions 1, 2 and 3, we assume that \( x^*_{iJSL} < x^*_i \) and we construct the following string of inequalities:

\[ 0 = \frac{\partial B^*_{iJSL}}{\partial x^*_i} - \frac{\partial B^*_{iJSL}}{\partial k^*_{iJSL}} + r_k c'(k^*_{iJSL}) - (1 - P_i^{*JSL}) \left( \frac{1}{X} s_j^{*JSL} L(X^{*JSL}) + s_i^{*JSL} \frac{\partial L}{\partial x_i^{*JSL}} \right) \]

\[ - \frac{\partial P_i}{\partial k^*_{iJSL}} \left( s_i + s_j P_j^{*JSL} \right) L(X^{*JSL}) \]

\[ \leq \frac{\partial B^*_{iJSL}}{\partial x^*_i} - \frac{\partial B^*_{iJSL}}{\partial k^*_{iJSL}} + r_k c'(k^*_{iJSL}) - (1 - P_i^{*JSL}) \left( s_i + s_j P_j^{*JSL} \right) \frac{\partial L}{\partial x_i^{*JSL}} \]

\[ = 0. \]

Contradiction.  ■