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Pricing of Complementary Goods and Network Effects

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* The Networks, Electronic Commerce, and Telecommunications (“NET”) Institute, <http://www.NETinst.org>, is a non-profit institution devoted to research on network industries, electronic commerce, telecommunications, the Internet, “virtual networks” comprised of computers that share the same technical standard or operating system, and on network issues in general.

Pricing of Complementary Goods and Network Effects*

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Nicholas Economides** and V. Brian Viard***

Abstract

We discuss the case of a monopolist of a base good in the presence of a complementary good provided either by it or by another firm. We assess and calibrate the extent of the influence on the profits from the base good that is created by the existence of the complementary good, *i.e.*, the extent of the network effect. We establish an equivalence between a model of a base and a complementary good and a reduced-form model of the base good in which network effects are assumed in the consumers' utility functions as a surrogate for the presence of direct or indirect network effects, such as complementary goods produced by other firms. We also assess and calibrate the influence on profits of the intensity of network effects and quality improvements in both goods. We evaluate the incentive that a monopolist of the base good has to improve its quality rather than that of the complementary good under different market structures. Finally, based on our results, we discuss a possible explanation of the fact that Microsoft Office has a significantly higher price than Microsoft Windows although both products have comparable market shares.

Key words: calibration; monopoly; network effects; complementary goods; software; Microsoft

JEL Classification Codes: L12; L13; C63; D42; D43

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Pricing of Complementary Goods and Network Effects

1. Introduction

We examine a monopolist of a base good who benefits from a complementary good provided either by it or another firm. We assess and calibrate the extent of the positive influence (network effect) on the base good profits that is created by the existence of the two sources (internal and/or external) of complementary goods. We establish an equivalence between a model of a base and complementary good and a reduced form model of the base good where network effects are assumed in the utility function as a surrogate for the presence of direct network effects (*i.e.*, a consumer's utility directly increases in the number of other users) or indirect network effects (*i.e.*, arising from increased variety of complementary goods produced by other firms). We assess and calibrate the influence of the intensity of network effects and quality improvements in the complementary good on profits from the base good. We also evaluate the incentive that a monopolist has to improve the quality of the base good rather than that of a complementary good that it produces.

Our model has implications for the base good monopolist's tradeoff in improving the quality of its own complementary good versus subsidizing increases in other network effects. The monopolist could subsidize increases in other network effects by, for example, taking actions to increase sales of the base good thereby increasing consumers' utility directly ("direct network effects") or facilitating or subsidizing increased variety of other complementary goods available ("indirect network effects"). The base good monopolist prefers that an independent firm offer an additional complementary good rather than improve the quality of a pre-existing complementary good by the same amount as the quality offered by the new good, assuming the costs of the two are the same. This results from the complementary goods firm's incentive to restrict output more at higher quality levels, limiting the increase in base good sales due to complementarities or indirect network effects. We also find that the base good monopolist gains more from adding a complementary good to its portfolio of products than increasing the quality of an existing portfolio product by the same quality as that of the new good if the costs of doing so are the same. The effect is stronger than if an independent firm produces the complementary good. This is because adding a complementary good increases sales of the base good because of the complementarities, but an increase in the quality of the complementary good does not affect sales of the base good because the monopolist can fully adjust the price of the complementary good to capture profits from its increased quality.

The model of this paper also has implications for the base good monopolist's incentives to invest in improving the base and complementary goods under different market structures and in making them compatible. An independent base good monopolist has a greater incentive to invest in improving the quality of the base good (at the margin) than a joint monopolist who produces both the base and complementary good. Improvements in the base good increase its price and therefore the effective price to use the complementary good. A producer of both internalizes this and has less incentive to improve the base good, while an independent monopolist does not. The flip side of this is that a monopolist who produces both the base and complementary goods has a greater incentive to improve the complementary good (on the

margin) than an independent firm would have to improve it. Improvements in the complementary good's quality increase sales of the base good, which the joint monopolist internalizes but the independent firm does not. Finally, if a single firm owns both the base and complementary goods it has a greater incentive to make them compatible than if separate firms offered the two products because increasing compatibility improves sales of both. The base good benefits directly from a more compatible product and the complementary good benefits indirectly because it requires purchase of the base good. A joint monopolist internalizes this feedback while independent firms do not.

Based on our results, we discuss a possible explanation of the fact that Microsoft Office is significantly more expensive than Microsoft Windows. Microsoft has approximately the same market share (over 90%) in the market for operating systems for personal computers as in the market for "office applications" (a bundle of word processing, spreadsheet, presentation and database software). However, Microsoft charges a price for its Windows operating system that is significantly lower than the price of its office suite. Although our model does not address the level of prices for Windows and Office, it can explain this difference in relative prices for Windows and Office. A joint monopolist, such as Microsoft, has two price instruments, the base good and complementary good price. It is optimal to keep the operating system price low even if Office is quite valuable because some users buy Windows for use with other complementary goods. Raising the price of Office but keeping the price of Windows low allows the joint monopolist to capture some of the value provided by Office while not pricing users of other complementary goods out of the market.

In the extensive literature on network effects, there are two types of models: those that attempt to derive the network effect from the detailed microeconomics of the model and those that assume the existence of network effects and discuss the consequences for market structure. The first approach has been called the "micro approach" to network economics, while the second one has been called the "macro approach."^{1, 2} In the macro approach, network effects are typically summarized by a term that influences utility positively and is increasing in sales. The macro approach derives from Katz and Shapiro (1985) and Farrell and Saloner (1986), while the micro approach originated from Matutes and Regibeau (1988) and Economides (1989). There has been little attempt to calibrate the size of the network effect used in the macro approach models. Two exceptions are Economides (1996b) and Clements (2004). Economides (1996b) calibrates the size of the network effect in the context of measuring the incentive of a patent-holding monopolist who also sells a complementary good to invite competitors in the complementary goods market so as to maximize the network effects. Unlike our paper, Clements (2004) evaluates the effect of the strength of network effects, degree of compatibility and the density of consumers in the market on standardization under oligopolistic competition.

¹ See Economides (1996a).

² Shy (2001) calls these the "components approach" and the "network externalities approach." We do not consider the more distantly related effect of changes in "software variety" considered in Church and Gandal (1992) or Chou and Shy (1990). The former evaluate how the compatibility decisions of software firms affect the degree of standardization in the hardware market, while the latter demonstrate that increasing returns in the production of complementary goods can substitute for the assumption of network effects. These are different than our objective, which is to take network effects and complementary goods as given and evaluate their equivalence.

This is a different objective than ours of providing an equivalence of the two modeling approaches and the resulting implications.

Micro models include both models of “mix and match,” where consumers assemble systems in fixed proportions, and “indirect network effects,” where consumption benefits arise from consuming systems and where production of the complementary good is characterized by increasing returns to scale.³ Our model is closer to the former since we focus on a single complementary good, but differs from typical assumptions in “mix and match” models because the base good in our model is valuable without use of the complementary good whereas in “mix and match” models typically neither good is valuable without the other. In our micro model, there are several ways in which network effects could arise from the base good monopolist’s offering a complementary good. For example, there could be direct network effects (such as file-sharing across users) in the complementary good or there could be increasing returns in producing the complementary good.

The network effects summarized in the macro models can result from either direct or indirect network effects. In the former case, a consumer benefits directly from the number of individuals adopting the base good (for example, because there is a larger network to communicate with) while in the latter case a consumer benefits indirectly from the number of individuals adopting the base good through the increased availability of software variety. Both of these effects are summarized by a term in consumers’ utility functions, which is increasing in total sales of the base good.

Section 2 sets up the basic framework of our research. Section 3 develops and discusses the five models we use in this paper, which differ in the way that network effects and inherent product quality are modeled. Section 4 compares the equilibria of the five models. Section 5 discusses the incentives to invest in quality in either the base good or the complementary good in different ownership structures and under different intensities of network effects and also examines compatibility decisions made by the base good monopolist. Section 6 discusses the explanation of Microsoft’s relative pricing provided by our analysis. Section 7 compares our results with the empirical literature on network effects. Section 8 has concluding remarks.

2. Basic Framework

We assume that consumers are differentiated in terms of their preferences for quality of the base good (“B”) and quality of the complementary good (“C”). The second good requires the first good to provide positive utility.⁴ For example, we can think of the Windows operating system as the base good, and an office suite (such as Microsoft Office) as the complementary good, not necessarily produced by the same company. Let the marginal utility of quality of the

³ See, for example, Chou and Shy (1990).

⁴ Since the complementary good requires the presence of the base good but not conversely, we expect that the equilibria in terms of prices and quantities will be asymmetric across firms.

base good be θ and the marginal utility of quality of the complementary good be φ . The pair (θ, φ) defines a consumer type. We assume that both θ and φ are distributed independently and uniformly on $[0, 1]$.

We assume that, besides the complementary good that we explicitly model, there are potentially other network effects for the base good (“additional network effects”). These additional network effects could be direct network effects that result from a consumer’s utility directly increasing in the number of other users of the base good or indirect network effects that result from other complementary goods whose existence positively influences consumers’ willingness to pay for the base good. We assume in the latter case that the positive consumption effects between the base good and the complementary goods reinforce each other. We summarize these effects by adding a term proportional to sales of the base good in the utility function of a typical consumer.

When consuming one unit of the base good and possibly one unit of the complementary good, consumer (θ, φ) receives utility

$$U = k_B + \theta q_B - p_B + \alpha x_B + \delta V,$$

where k_B is a constant, q_B is the quality of the base good, p_B is the price of the base good, V is the utility from the consumption of the complementary good, x_B is the sales of the base good, α measures the intensity of the additional network effects and δ is an indicator variable taking the value one if the complementary good is bought and zero otherwise. Thus, network effects arising out of direct or indirect network effects from other complementary goods are summarized by an additive term in the utility function proportional to sales.⁵ The utility from the consumption of the complementary good is

$$V = k_C + \varphi q_C - p_C,$$

where k_C is a constant, q_C is the quality of the complementary good and p_C is the price of the complementary good.

We will consider five alternative models. The first model has a base good monopolist in a market where additional network effects are summarized in the utility function of consumers as proportional to sales. The second model has two monopolists (independent firms), one for the base good and one for the complementary good, and assumes no additional network effects. The third model adds additional network effects to the independent firms in Model 2. The fourth model has a single monopolist (joint monopolist) producing both the base and the complementary good. The fifth model adds additional network effects to the joint monopolist considered in Model 4.

⁵ We assume that that the influence of positive consumption (network) effects on the willingness to pay for the base good can be summarized by an additive term which is proportional to sales of the base good. This assumes that higher sales of the base good are reflected in higher sales of the complementary good and vice versa.

3. Models

3.1 Model 1: Single Good Monopolist in a Market with Additional Network Effects

We first consider a model of a single good monopolist selling the base good with additional network effects arising from direct network effects or from indirect network effects due to the presence of other complementary goods. In this case, $\delta = 0$ and consumer θ who buys one unit of the base good of quality q_B at price p_B receives utility of

$$U = \theta q_B - p_B + \alpha x_B \quad (1)$$

where $\alpha > 0$ measures the intensity of the additional network effect (marginal utility of network expansion).⁶ All consumers of type $\theta > \theta_B$ buy the good, where the marginal consumer is

$$\theta_B = (p_B - \alpha x_B)/q_B. \quad (2)$$

Sales are

$$x_B = (1 - \theta_B) = 1 - (p_B - \alpha x_B)/q_B. \quad (3)$$

Inverting the demand we have

$$x_B = (q_B - p_B)/(q_B - \alpha), \quad \Pi_B = p_B x_B = p_B(q_B - p_B)/(q_B - \alpha).^7 \quad (4)$$

Assuming zero costs, maximizing profits implies:⁸

$$p_B^* = q_B/2, \quad x_B^* = q_B/(2(q_B - \alpha)) \text{ and } \Pi_B^* = q_B^2/(4(q_B - \alpha)).^9 \quad (5)$$

In the case of no additional network effects, *i.e.*, when $\alpha = 0$, the demand without network effects is a pivot of the demand with network effects through the point $(0, q_B)$. It is well known that such pivots of linear demands lead to the same monopoly price. Thus, the equilibrium price is unaffected by additional network effects, while sales and profits are higher with them. Using the subscript 0 for the variables with no additional networks effects ($\alpha = 0$), we have

$$x_B^* = x_{B0}[q_B/(q_B - \alpha)], \quad p_B^* = p_{B0} \text{ and } \Pi_B^* = \Pi_{B0}[q_B/(q_B - \alpha)]. \quad (6)$$

⁶ In sections 3 through 5, in which we focus on the theoretical model, we set $k_B = k_C = 0$ for simplicity.

⁷ We require $\alpha < q_B$ so that the demand is downward sloping.

⁸ We present the model with zero costs, but positive costs could easily be added. We have $d\Pi_B/dp_B = (q_B - 2p_B)/(q_B - \alpha) = 0$ and $d^2\Pi_B/dp_B^2 = -2/(q_B - \alpha) < 0$ since $q_B > \alpha$.

⁹ We also require that everyone does not buy the good which implies $x_B^* < 1$ or $2(q_B - \alpha) > q_B$, *i.e.*, $\alpha < q_B/2$.

3.2 Model 2: Independent Firms Without Additional Network Effects

In Model 2, we consider two independent monopolists, one for the base good and another for the complementary good, and we assume no additional network effects. By comparing the equilibrium of this model to that of Model 1, we can calibrate the intensity of network effects for the base good generated by sales of a complementary good.

There are two groups of purchasers to consider (see Figure 1). First, consumers of type $\theta > \theta_B$, $\varphi < \varphi_{B,BU}$ buy the base good only, where θ_B is the marginal consumer indifferent between buying the base good and buying nothing, *i.e.*

$$\theta_B = p_B / q_B, \quad (7)$$

and $\varphi_{B,BU}$ is the marginal consumer indifferent between buying only the base good and buying both the base and complementary goods, *i.e.*

$$\varphi_{B,BU} = p_C / q_C. \quad (8)$$

Second, consumers of types $\varphi > \varphi_{B,BU}$, $\theta > \theta_B$, as well as of types $\varphi > \varphi_{BU}(\theta)$, $\theta < \theta_B$, buy both, where $\varphi_{BU}(\theta)$ is the marginal consumer of type θ indifferent between buying both goods and buying nothing, *i.e.*

$$\varphi_{BU}(\theta) = (p_B + p_C - \theta q_B) / q_C. \quad (9)$$

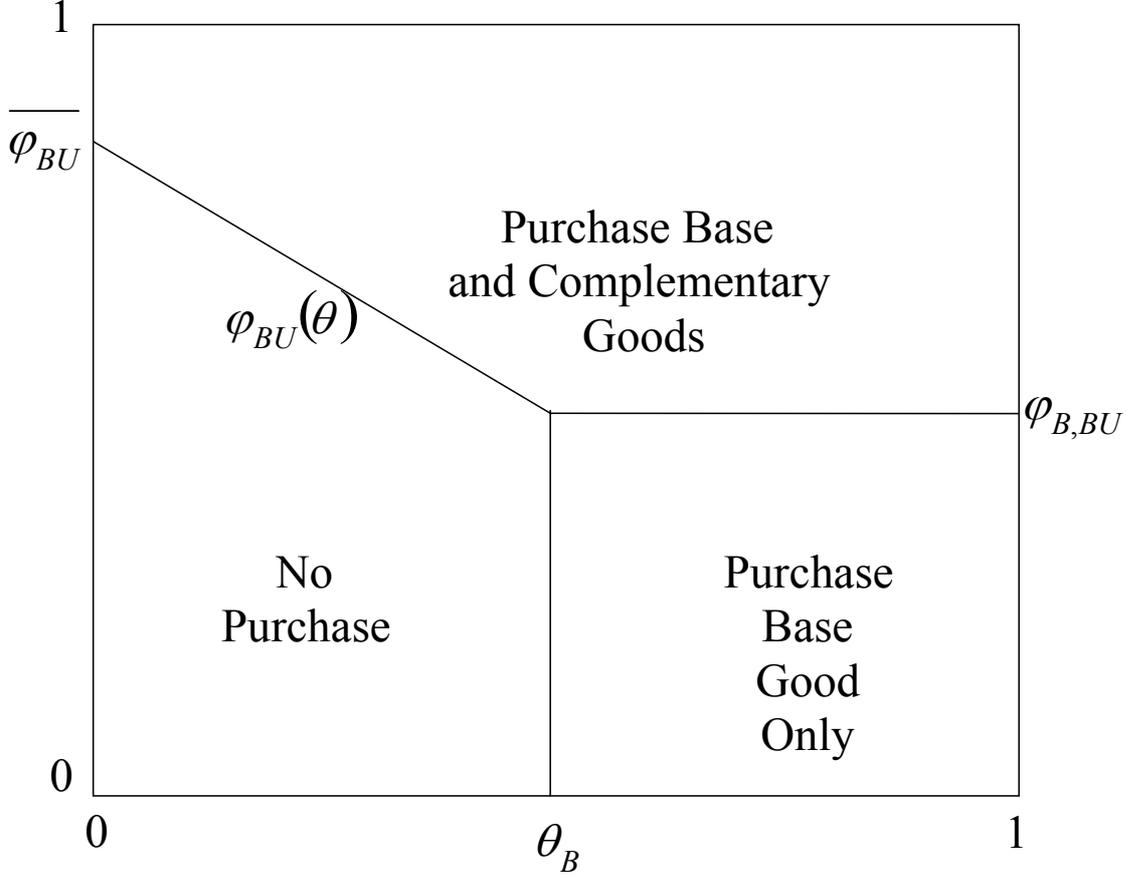
The profits for the base good monopolist are

$$\Pi_B = [(1 - \theta_B)\varphi_{B,BU} + (1 - \varphi_{B,BU} - (\overline{\varphi_{BU}} - \varphi_{B,BU}) / 2)] p_B, \quad (10)$$

where $\overline{\varphi_{BU}} = (p_B + p_C) / q_C$ is the consumer of type $\theta = 0$ who is indifferent between buying both goods and nothing. The profits for the complementary good monopolist are

$$\Pi_C = [(1 - \varphi_{B,BU}) - (\overline{\varphi_{BU}} - \varphi_{B,BU}) \theta_B / 2] p_C. \quad (11)$$

Figure 1



At a Nash equilibrium in a price-setting game, the first-order conditions for the two monopolists are

$$p_B(3p_B + 4p_C) - 2q_Bq_C = 0 \text{ and } p_B^2 - 2q_B(q_C - 2p_C) = 0. \quad (12)$$

Since the first-order conditions are nonlinear we solve them numerically to find the equilibrium.¹⁰ In our analysis we will restrict $q_C \geq q_B$ since we wish to consider interior solutions only and for $q_C > q_B$, $\overline{\varphi_{BU}} > 1$.¹¹

¹⁰ The second order conditions are $-(3p_B + 2p_C)/q_Bq_C < 0$ and $-2/q_C < 0$ respectively, both of which are met for all parameter values.

¹¹ Note that the first-order conditions themselves place no restrictions on the relative qualities. The positive root of the first first-order condition is $1/3(-2p_C + \sqrt{4p_C^2 + 6q_Bq_C})$ which is always positive. Solving the second first-order condition for p_C we get $q_C/2 + p_B^2/4q_B$ which is always positive as well.

3.3 Model 3: Independent Firms With Additional Network Effects

In Model 3, we incorporate additional network effects arising from other complementary goods into Model 2, in addition to the effects of the complementary good already in that model. The utility function of consumers (equation (1)) now has a positive α capturing the additional network effects. The same regions of consumer types buy as in Model 2, but some margins now depend on α . We use superscript n to denote the presence of additional network effects

$$\theta_B^n = (p_B - \alpha x_B)/q_B, \quad \varphi_{B,BU}^n = \varphi_{B,BU}, \quad (13)$$

$$\varphi_{BU}^n = (p_B + p_C - \theta q_B - \alpha x_B)/q_C, \quad \overline{\varphi_{BU}^n} = (p_B + p_C - \alpha x_B)/q_C. \quad (14)$$

Demand for the base good is given by solving for x_B in

$$x_B = 1 - \theta_B^n(x_B) + \left(1 - \left(\overline{\varphi_{BU}^n}(x_B) - \varphi_{B,BU}^n(x_B)\right)\theta_B^n/2\right). \quad (15)$$

Since θ_B^n and φ_{BU}^n are both linear functions of x_B , this is a quadratic equation. Using the positive root, x_B^* , that solves this equation, the profit function for the base good monopolist is

$$\Pi_B^n = \left[\left(1 - \theta_B^n(x_B^*)\right)\varphi_{B,BU}^n + \left(1 - \varphi_{B,BU}^n - 1/2\left(\overline{\varphi_{BU}^n}(x_B^*) - \varphi_{B,BU}^n\right)\theta_B^n(x_B^*)\right) \right] p_B \quad (16)$$

and for the complementary good monopolist is

$$\Pi_C^n = \left[\left(1 - \varphi_{B,BU}^n\right) - 1/2\left(\overline{\varphi_{BU}^n}(x_B^*) - \varphi_{B,BU}^n\right)\theta_B^n(x_B^*) \right] p_C. \quad (17)$$

The first-order conditions for the two firms are nonlinear functions of the prices so we solve them numerically.¹²

3.4 Model 4: Joint Monoplist Without Additional Network Effects

In Model 4, the joint monopolist sells both the base and complementary goods. The marginal consumers are defined in the same manner as in Model 2, and the profit function for the joint monopolist is

$$\Pi_B + \Pi_C = (1 - \theta_B)\varphi_{B,BU}p_B + \left(1 - \varphi_{B,BU} - 1/2\left(\overline{\varphi_{BU}} - \varphi_{B,BU}\right)\theta_B\right)(p_B + p_C). \quad (18)$$

¹² We also verify numerically that the nonlinear second-order conditions hold and that $\Pi_B^n(p_B; p_C^*)$ is quasiconcave in p_B and $\Pi_C^n(p_C; p_B^*)$ is quasiconcave in p_C .

The joint monopolist chooses both prices to maximize its profits. The first-order conditions are

$$3p_B(p_B + 2p_C) - 2q_Bq_C = 0 \text{ and } 3p_B^2 - 2q_B(q_C - 2p_C) = 0. \quad (19)$$

These can be solved to get the equilibrium prices, quantities, and profits:

$$p_B = 2q_B/3, \quad p_C = q_C/2 - q_B/3, \quad (20)$$

$$x_B = 1 - p_B(p_B + 2p_C)/(2q_Bq_C) \text{ and } x_C = 1 - (p_B^2 + 2p_Cq_B)/(2q_Bq_C).^{13} \quad (21)$$

Notice that the price of the base good is independent of the quality of the complementary good. This is true for general demand functions, since the marginal revenue of the joint monopolist from sales of the base good is independent of the quality and price of the complementary good, at the optimal complementary good price.¹⁴ The joint monopolist completely internalizes in the complementary good price any changes in the quality of the complementary good, and therefore the price of the basic good remains unaffected by such quality changes.¹⁵ In our analysis we will only consider positive prices for the complementary good and therefore restrict $q_C > 2q_B/3$.¹⁶

3.5 Model 5: Joint Monopolist With Additional Network Effects

In Model 5, we incorporate additional network effects for the base good into Model 4. The marginal consumers are defined in the same manner as in Model 3 and the profit function for the joint monopolist is

¹³ The second-order condition is met as the Hessian is negative definite for all parameter values.

¹⁴ To see this in general, consider general demand functions for the base and complementary goods, respectively $D_B(p_B)$ and $D_C(p_B + p_C)$. Then profits are $\Pi_B = p_B[D_B(p_B) + D_C(p_B + p_C)]$, $\Pi_C = p_C D_C(p_B + p_C)$, and joint profits are $\Pi = \Pi_B + \Pi_C$ so that the first order conditions are (where primes denote derivatives):

$$(A) \quad D_B + p_B D_B' + D_C + (p_B + p_C) D_C' = 0,$$

$$(B) \quad D_C + (p_B + p_C) D_C' = 0,$$

which imply $D_B(p_B) + p_B D_B'(p_B) = 0$. Therefore for the joint monopolist the choice of price for the base good is independent of the choice of price and quality of the complementary good.

¹⁵ Also notice that, for independent firms, the first order conditions cannot be decomposed as in joint monopoly, and therefore the equilibrium prices of both the base and complementary good do depend on the quality levels of both goods. For independent firms, the first order conditions are:

$$(A') \quad D_B + p_B D_B' + D_C + p_B D_C' = 0,$$

$$(B') \quad D_C + p_C D_C' = 0.$$

Substitution from (B') into (A') cannot accomplish decomposition as in joint monopoly. Of course, comparison of (B) with (B') confirms Cournot's result that the total price $p_B + p_C$ is lower under joint monopoly.

¹⁶ Although in principle the joint monopolist could choose to sell the complementary good below cost, such action could raise serious antitrust concerns.

$$\Pi_B^n + \Pi_C^n = (1 - \theta_B^n(x_B^*)) \varphi_{B,BU}^n p_B + \left(1 - \varphi_{B,BU}^n - 1/2 \left(\overline{\varphi_{BU}^n(x_B^*)} - \varphi_{B,BU}^n \right) \theta_B^n(x_B^*) \right) (p_B + p_C). \quad (22)$$

The first-order conditions for the firm are nonlinear functions of the prices so we solve them numerically.¹⁷

4. Equivalence Results

In this section, we calibrate the size of network effects arising from sales of complementary goods. This is possible since we have models that explicitly allow for positive effects of complementary goods sales as well as models that allow for network effects that are summarized in the utility function. Thus, we establish an equivalence between the network effects (defined as added profits to a base good monopolist) created by the presence of a complementary good and those summarized in the utility function. This is done in sections 4.1 to 4.4 for the various industry structures and for different quality levels. We use this equivalence in base good profits to analyze the incentive of the base good monopolist to offer its own complementary good, improve the quality of a complementary good that it offers, and subsidize an independent firm so that it offers or increases the quality of a complementary good it provides.

We focus on a particular type of equivalence, in base good profits, because we are primarily interested in the incentives of the base good monopolist. This equivalence, of course, does not ensure that consumer welfare is equated. Doing so would require determining how to weight the utility of consumers with high versus low valuations of each of the base and complementary goods since purchasing patterns will vary across different equilibria. In addition, profits could not be equilibrated at the same time as consumer welfare. One could also calibrate equivalence in total profits across both firms. However this would be an inappropriate comparison to make when comparing models with and without additional network effects, as we do. Similarly we could calibrate the equivalence in total (base and complementary good) profits for the base good monopolist. However this would also be an inappropriate comparison to make when comparing models in which the base good firm controls the complementary good to those in which it does not. We focus on equivalence in base good profits because our goal is to evaluate the incentives of the base good monopolist.¹⁸

An important property of all our models is scalability. It is easy to check that the equilibrium sales x_B , x_C are unaffected by a common scaling up or down of q_B , q_C , and α by the same positive coefficient, say $\lambda > 0$. Additionally, the equilibrium prices p_B and p_C are proportional to the common scaling factor λ of q_B , q_C , and α , and therefore their ratio

¹⁷ We also verify that the second-order conditions are met. We solve over a grid of possible prices to ensure that we obtain the global maximum.

¹⁸ We do not discuss the possibility of anti-competitive “leveraging” of monopoly power from the base to the complementary good.

(p_C/p_B) is unaffected by scaling. It follows that equilibrium profits are also proportional in the scaling factor λ . Thus, we scale (normalize) all our variables in terms of the quality of the base good q_B , defining the “normalized quality” of the complementary good as $\tilde{q}_C = q_C/q_B$, the “normalized α ” or “normalized network effects” as $\tilde{\alpha} = \alpha/q_B$, the normalized prices of the two goods as $\tilde{p}_B = p_B/q_B$, $\tilde{p}_C = p_C/q_B$, the “normalized relative price of the complementary good” in relation to the base good as $\tilde{R}_{CB} = \tilde{p}_C/\tilde{p}_B = p_C/p_B$, the “normalized base good profits” as $\tilde{\Pi}_B = \Pi_B/q_B$, and the “normalized complementary good profits” as $\tilde{\Pi}_C = \Pi_C/q_B$. All the normalized variables remain unaffected by the common scaling up or down of q_B , q_C , and α . Below, we will report results for all models in terms of these normalized variables.

4.1 Equivalence Between Additional Network Effects And The Effects Of A Complementary Good Produced By An Independent Firm (Model 1 Versus Model 2)

We start with a model of two independent monopolists, one producing the base good and another producing a complementary good (Model 2). We compare this with a model of a single base good monopolist where the benefit of complementary goods to consumers is summarized in their utility function (Model 1). We establish an equivalence between the two models by equating the normalized base good equilibrium profits. An independent firm selling the complementary good results in increased sales of the base good. Such network effects can be alternatively represented by “additional network effects” (αx) in the utility of individual consumers, where α measures the intensity of the network effect. Table 1 shows the normalized network effects, $\tilde{\alpha}$, required to obtain equivalent normalized base good profits in the absence of the complementary good. For example, line three of the table indicates that a base good monopolist in the absence of a complementary good but with an $\tilde{\alpha}$ of 0.4149 earns the same normalized base good profits as a base good monopolist with an $\tilde{\alpha}$ of zero in the presence of an independent monopolist producing a complementary good of normalized quality $\tilde{q}_C = 3$. In this and all following analyses we choose $q_C \geq q_B$ to ensure an interior solution for the independent firms market structure and to ensure a positive price for the complementary good in the joint monopolist market structure as described earlier.

Table 1 Independent Firms: Equivalence of Quality and Network Effects*

Normalized Complementary Good Quality $\left(\tilde{q}_C = \frac{q_C}{q_B}\right)$	Normalized Relative Price of Complementary Good $\left(\tilde{R}_{CB} = \frac{p_C}{p_B}\right)**$	Normalized Base Good Profits $\left(\tilde{\Pi}_B = \frac{\Pi_B}{q_B}\right)$	Normalized Equivalent α $\left(\tilde{\alpha} = \frac{\alpha}{q_B}\right)$
1	0.7071	0.3431	0.2714
2	1.2291	0.4003	0.3755
3	1.7375	0.4273	0.4149
5	2.7441	0.4532	0.4484
10	5.2481	0.4755	0.4743

* In this and all subsequent tables, we round results to four decimal places unless otherwise noted.

** These are equilibrium prices under presence of the complementary good but no additional network effects.

4.2 Equivalence Between Additional Network Effects And The Effects Of A Complementary Good Produced By The Joint Monopolist (Model 1 Versus Model 4)

The joint monopolist's sales of the base good increase when it also sells the complementary good. We find the normalized network effects, $\tilde{\alpha}$, required to obtain equivalent normalized base good profits by a monopolist providing only the base good. The results are summarized in Table 2. This is equivalent to Table 1 but for a joint monopolist rather than for two independent firms. For example, line three of the table indicates that a monopolist producing a base good in the absence of a complementary good with an $\tilde{\alpha}$ of 0.4375 earns the same normalized base good profits as a monopolist selling a base and a complementary good of normalized quality $\tilde{q}_C = 3$ with an $\tilde{\alpha}$ of zero.

Table 2 Joint Monopolist: Equivalence of Quality and Network Effects

Normalized Complementary Good Quality $\left(\tilde{q}_C = \frac{q_C}{q_B}\right)$	Normalized Relative Price of Complementary Good $\left(\tilde{R}_{CB} = \frac{p_C}{p_B}\right)^*$	Normalized Base Good Profits $\left(\tilde{\Pi}_B = \frac{\Pi_B}{q_B}\right)$	Normalized Equivalent α $\left(\tilde{\alpha} = \frac{\alpha}{q_B}\right)$
1	0.2500	0.4444	0.4375
2	1.0000	0.4444	0.4375
3	1.7500	0.4444	0.4375
5	3.2500	0.4444	0.4375
10	7.0000	0.4444	0.4375

* These are equilibrium prices under presence of complementary good but no additional network effects.

The results in Table 2 are presented in numerical form for easy comparisons with other tables. They can also be presented in algebraic form using equations (20) – (21) as: $\tilde{R}_{CB} = 3\tilde{q}_C/4 - 1/2$ and $\tilde{\Pi}_B = 4/9$. Equating $\tilde{\Pi}_B$ to the normalized base good profits from Model 1 (a single good monopolist with additional network effects) gives (from equation (5)) the equivalent $\tilde{\alpha}$ of $\tilde{\alpha} = 7/16$.

Comparing Tables 1 and 2, we observe that, while normalized base good profits are sensitive to the normalized quality of the complementary good for the independent monopolist, they are not for the joint monopolist. For the joint monopolist, all the variation in the normalized quality of the complementary good is reflected in the normalized complementary good price and the normalized base good price is unaffected (*i.e.*, $\tilde{p}_B = 2/3$ while $\tilde{p}_C = (\tilde{q}_C/2) - 1/3$).¹⁹ This follows from the fact that the joint monopolist is able to adjust the price of the complementary good to fully reflect its adjustment in quality. Since it has both price instruments available, the joint monopolist can adjust the complementary good price so that the margin for consumers buying only the base good (the θ_B margin in Figure 1) is not distorted by the change in complementary good quality. The joint monopolist does not want to alter the base good price because consumers who buy only the base good may be priced out of the market since they do not benefit from complementary good quality improvements. In contrast, the independent monopolist of the base good, in a Nash equilibrium framework, changes its price in the direction of changes in the quality of the complementary good. Thus, base good prices and profits are sensitive to quality changes of the complementary good when independent firms produce the two goods separately but not when the same firm produces them. As a result, the strength of the additional network effects (as measured by the $\tilde{\alpha}$ needed to equate the normalized base good

¹⁹ As shown in general in footnote 14, the sum of the prices $p_B + p_C$ is lower for the joint monopolist than for the independent monopolists.

profits) is not sensitive to changes in the normalized quality for the joint monopolist but is for the equilibrium of independent firms.²⁰

We can use Tables 1 and 2 to assess the base good monopolist's incentive to invest in improving the quality of the complementary good when it owns the complementary good versus when an independent firm owns it. Microsoft in effect subsidizes compatible applications that it does not sell by including in Windows various functions that are useful to applications developers and which applications developers would have to develop by themselves if they were not available in Windows.²¹ As noted, the base good monopolist does not benefit (in terms of base good profits) from improvements in the complementary good quality when it owns both products, but does when an independent firm controls the complementary good. Therefore, the base good monopolist has a greater incentive to invest in improving the complementary good when an independent firm controls it. Of course, as a joint monopolist, there might be separate incentives provided by the complementary good profits that it would benefit from.

4.3 Equivalence Between A Low Quality Good With Additional Network Effects And A High Quality Good For An Independent Firm (Model 2 Versus Model 3)

We next analyze the effect of increasing the normalized quality of the complementary good when independent firms produce the base and complementary goods. We find the increase in the degree of normalized network effects that is equivalent to an increase in the normalized quality of the complementary good. In particular, we compare increases in the normalized quality of the complementary good in Model 2 to an increase in normalized network effects (an increase in $\tilde{\alpha}$, starting from 0) in Model 3 with a fixed normalized quality of $\tilde{q}_C = 1$. The results are summarized in Table 3. For example, line three of the table considers a base good monopolist in the presence of an independent complementary good monopolist with normalized relative quality $\tilde{q}_C = 1$. If the normalized quality of the complementary good is increased to $\tilde{q}_C = 5$ this is equivalent (in normalized base good profits) to increasing $\tilde{\alpha}$ from zero to 0.2792.

²⁰ We also observe that the $\tilde{\alpha}$'s are neither consistently higher nor lower for the joint monopolist relative to the independent firms. At high levels of complementary good quality the independent firms' $\tilde{\alpha}$ -equivalent is greater, while at low quality levels the opposite is true.

²¹ For example, Windows has timing functions that are useful to applications developers and have no direct functional value to end-users and built-in abilities to print to a variety of printers, a necessary capability for applications. Also note that all modern computer operating systems contain a variety of functions that are useful to applications developers but typically not directly useful to end-users.

Table 3 Independent Firms: Equivalence of Quality Increases and Network Effects

Increase in Normalized Complementary Good Quality $\left(\tilde{q}_C = \frac{q_C}{q_B}\right)$	Normalized Relative Price of Complementary Good (at High Quality)* $\left(\tilde{R}_{CB} = \frac{p_C}{p_B}\right)$	Normalized Base Good Profits (at High Quality)* $\left(\tilde{\Pi}_B = \frac{\Pi_B}{q_B}\right)$	Equivalent Increase in Normalized α $\left(\tilde{\alpha} = \frac{\alpha}{q_B}\right)**$
1 → 2	1.2291	0.4003	0.1591
1 → 3	1.7375	0.4273	0.2230
1 → 5	2.7441	0.4532	0.2792
1 → 10	5.2481	0.4755	0.3242

* These are equilibrium prices and profits under the higher normalized complementary good quality.

** Increase from zero.

We can also use these results to assess the incentive of the base good monopolist to subsidize an increase in the quality of an independent firm's complementary good versus subsidizing an additional complementary good offered by an independent firm. As we have seen, an independent monopolist who produces the base good has normalized profits of 1/4 when there is no complementary good and no additional network effects (Model 1). So an independent monopolist producing a base good in the absence of a complementary good and with no additional network effects earns normalized base good profits of 0.25. We can see from row one of Table 1 that a base good monopolist in the presence of an independent complementary good monopolist offering normalized complementary good quality $\tilde{q}_C = 1$ earns base good profits of 0.3431. Thus, adding one complementary good of quality $\tilde{q}_C = 1$ increases normalized base good profits by approximately 0.0931. This is larger than the normalized base good profits increase precipitated by a normalized quality increase from $\tilde{q}_C = 1$ to 2 in the complementary good (which, by comparing the base good profits in row one of Table 3 to base good profits in row one of Table 1, is approximately 0.0572).

Thus, a monopolist of the base good prefers that the independent firm add a complementary good of normalized quality 1 rather than increase the normalized quality of a complementary good from 1 to 2 if the costs of both changes are the same. Adding a complementary good expands the market for the base good more than an equivalent increase in the normalized quality of the complementary good increases the market for the base good because the elasticity of demand for the complementary good is declining in its quality so that the producer of the complementary good restricts output more per incremental increase in quality as the quality rises. This can be seen by computing the elasticity of demand for the complementary good from equation (11), which yields $-p_C/(q_C - p_C - 0.5\theta_B p_B)$.²² Thus at increasingly higher quality levels of the complementary good, the complementary good firm restricts output more and the base good monopolist benefits less from market expansion of the

²² The demand equation for the complementary good firm simplifies to $x_C = 1 - p_C/q_C - \theta_B p_B/2q_C$.

base good. The complementary goods firm does not internalize the profits from the market expansion of the base good and at increasingly higher quality levels the market for the complementary good becomes saturated (the marginal consumer approaches $\theta = 0$) so it is optimal to raise price and limit output. This may explain the behavior of Microsoft in subsidizing a greater number of applications that are independent goods with respect to each other (neither substitutes nor complements to each other) but are complementary goods to its Windows operating system (base good) rather than a few such applications of higher quality.

4.4 Equivalence Between A Low Quality Good With Additional Network Effects And A High Quality Good For A Joint Monopolist (Model 4 Versus Model 5)

In this section we analyze the effects of increasing the normalized quality of the complementary good when a joint monopolist produces the base and complementary goods. This is similar to the analysis reported in Section 4.3, but for the joint monopolist. We compare increases in the normalized quality in Model 4 to increases in normalized network effects (an increase in $\tilde{\alpha}$, starting from 0) in Model 5 with fixed normalized quality equal to $\tilde{q}_C = 1$. The results are reported in Table 4. For example, row three of Table 4 considers a joint monopolist producing base good and complementary good of normalized quality $\tilde{q}_C = 1$. If the normalized quality of the complementary good is increased to $\tilde{q}_C = 5$ no increase in $\tilde{\alpha}$ is required to maintain the same normalized base good profits. The base good profits are invariant to the complementary good quality. As noted earlier, the joint monopolist can adjust the price of the complementary good to fully reflect changes in its quality so it does not need to change the base good price. The zero $\tilde{\alpha}$ increases in Table 4 mean that the joint monopolist does not get any benefits in its normalized base good profits from increases in the normalized quality of the complementary good that it produces.

Table 4 Joint Monopolist: Equivalence of Quality Increases and Network Effects

Increase in Normalized Complementary Good Quality $\left(\tilde{q}_C = \frac{q_C}{q_B}\right)$	Normalized Relative Price of Complementary Good (at High Quality)* $\left(\tilde{R}_{CB} = \frac{p_C}{p_B}\right)$	Normalized Base Good Profits (at High Quality)* $\left(\tilde{\Pi}_B = \frac{\Pi_B}{q_B}\right)$	Equivalent Increase in Normalized α $\left(\tilde{\alpha} = \frac{\alpha}{q_B}\right)**$
1 → 2	1.0000	0.4444	0.0000
1 → 3	1.7500	0.4444	0.0000
1 → 5	3.2500	0.4444	0.0000
1 → 10	7.0000	0.4444	0.0000

* These are equilibrium prices and profits under the higher normalized complementary good quality.

** Increase from zero.

Comparing Tables 3 and 4, we observe that normalized profits for the base good are sensitive to improvements in the normalized quality of the complementary good for the

independent firms but not for the joint monopolist. As we discussed earlier, this follows from the fact that the joint monopolist is able to adjust the price of the complementary good to fully reflect its change in quality. In contrast, the independent monopolist of the base good, at the Nash equilibrium, changes its price in the direction of changes in the quality of the complementary good. Thus, to improve base good profits the joint monopolist should subsidize independent complementary goods and not its own, while an independent firm producing the base good benefits from both.

We can also use Tables 2 and 4 to assess the incentive of the joint monopolist to invest in increasing the quality of its complementary good versus adding a complementary good. From Model 1 we know that a monopolist producing only a base good and with no additional network effects earns normalized base good profits of $1/4$. So a monopolist producing a base good in the absence of a complementary good and with no additional network effects earns normalized base good profits of 0.25. We can see from row one of Table 2 that a joint monopolist offering a base good along with a complementary good of normalized quality $\tilde{q}_C = 1$ earns normalized base good profits of $4/9$. Thus, adding one complementary good of normalized quality $\tilde{q}_C = 1$ increases normalized base good profits by $7/36$. From row one of Table 4 we see that increasing the normalized quality level of the complementary good has no effect on normalized base good profits. As discussed in Section 4.2, this is because the joint monopolist has both price instruments available and can adjust the complementary good price optimally without distorting the margin for consumers buying only the base good. This implies that the joint monopolist has an incentive to add a complementary good of minimal quality but not invest in its improvement based on its effect on base good profits only.²³

5. Effect of Quality Levels And Additional Network Effects On Profits

An important question frequently posed in the network effects literature concerns the incentive to improve the quality of products and how this is affected by the presence of complementary goods and network effects. In this section we assess the incentive for firms to invest in quality at the margin under different market structures (joint monopoly versus independent firms) and different levels of additional network effects. Although we do not explicitly model an investment stage we can assess the incentives to invest at the margin by considering the marginal effects of quality improvements on base good profits.

We first assess the effect on profits of quality changes in the base and complementary goods in the presence of varying levels of additional network effects. We also contrast the

²³ We could evaluate the base good monopolist's incentives based on total rather than marginal profits but this would require specifying the cost structure for quality improvements. The results would thus depend arbitrarily on the functional form of the cost function. We could also evaluate the base good monopolist's incentives based on base and complementary goods profits rather than just base good profits. However, our objective is instead to assess the base good monopolist's incentive beyond that provided by complementary good profits alone since these incentives would be the same for both a joint monopolist and an independent firm facing the same cost structure and in the absence of complementarities.

effects of quality changes when independent firms produce the two products to those when a joint monopolist produces both.

We first look at the effect of changes in base good quality on the normalized base good profits of the base good monopolist with an independent monopolist providing a complementary good. The results on $\frac{d\tilde{\Pi}_B}{dq_B}$ are reported in column 1 of Table 5 for different combinations of normalized network effects and normalized complementary good quality levels.²⁴

Second, we assess the effect of changes in the complementary good quality on the normalized base good profits of the base good monopolist with an independent monopolist providing a complementary good. The results on $\frac{d\tilde{\Pi}_B}{dq_C}$ are reported in column 2 of Table 5.

Third, we assess the effect of changes in the base good quality on the normalized complementary good profits of the complementary good monopolist with an independent monopolist providing the base good. The results on $\frac{d\tilde{\Pi}_C}{dq_B}$ are reported in column 3 of Table 5.

Fourth, we assess the effect of changes in the complementary good quality on the normalized complementary good profits of the complementary good monopolist with an independent monopolist providing the base good. The results on $\frac{d\tilde{\Pi}_C}{dq_C}$ are reported in column 4 of Table 5.

Fifth, we look at the effect of changes in base good quality on the normalized base good profits (Column 5) and normalized complementary good profits (Column 7) of the joint monopolist. The total effect of the change in base good quality on the joint monopolist is the sum of the two columns.

²⁴ Derivatives are calculated using Richardson extrapolation (see, for example, Acton, F. S. *Numerical Methods that Work*, 2nd printing, Washington, D.C., Mathematics Association of America, 1990, page 106) with a step-size of one percent of q_B or q_C .

Sixth, we look at the effect of changes in complementary good quality on the normalized base good profits (Column 6) and normalized complementary good profits (Column 8) of the joint monopolist. Again, the total effect of the change in the complementary good quality on the joint monopolist is the sum of the two columns.

Each row of table 5 provides these six effects at a given combination of normalized network effects and normalized quality levels. For example, row two shows that for an $\tilde{\alpha}$ of zero and $\tilde{q}_C = 2$, a marginal increase in base good quality increases the normalized profits of the independent base good monopolist by 0.3270, decreases the normalized profits of the independent complementary good monopolist by 0.0571, increases the joint monopolist's normalized base good profits by 0.4444 and decreases the joint monopolist's normalized complementary good profits by -0.1482. At the same level of normalized network effects and quality levels, a marginal increase in complementary good quality increases the normalized profits of the independent base good monopolist by 0.0366, normalized profits of the independent complementary good monopolist by 0.2194, increases the joint monopolist's normalized complementary good profits by 0.2593 and does not affect the joint monopolist's normalized base good profits.

Note that increasing the quality of the base good decreases normalized profits for the complementary good $\left(\frac{d\tilde{\Pi}_C}{dq_B} < 0\right)$, when independent monopolists produce the two goods. Since the base good is required for consumers to value the complementary good, an increase in the base good's quality increases price sufficiently that the complementary good firm's profits are squeezed. On the other hand, $\frac{d\tilde{\Pi}_B}{dq_C} > 0$ when an independent monopolist produces the second good. Improving the complementary good increases complementary good consumers' willingness to pay for the base good, which complementary good consumers must purchase, thus allowing the base good monopolist to increase the price of the base good. Because of the increased base good price, the monopolist loses some sales to base-good-only consumers, but not enough to offset the increased revenues from complementary good consumers.

Table 5 Effects of Quality Increases on Profits (“IF” = Independent Firms, “JM” = Joint Monopolist)

		1	2	3	4	5	6	7	8
Normalized α $\left(\tilde{\alpha} = \frac{\alpha}{q_B}\right)$	Normalized Complementary Good Quality $\left(\tilde{q}_C = \frac{q_C}{q_B}\right)$	IF	IF	IF	IF	JM	JM	JM	JM
		$\frac{d\tilde{\Pi}_B}{dq_B}$	$\frac{d\tilde{\Pi}_B}{dq_C}$	$\frac{d\tilde{\Pi}_C}{dq_B}$	$\frac{d\tilde{\Pi}_C}{dq_C}$	$\frac{d\tilde{\Pi}_B}{dq_B}$	$\frac{d\tilde{\Pi}_B}{dq_C}$	$\frac{d\tilde{\Pi}_C}{dq_B}$	$\frac{d\tilde{\Pi}_C}{dq_C}$
0	1	*	*	*	*	0.4444	0.0000	-0.1852	0.2870
0	2	0.3270	0.0366	-0.0571	0.2194	0.4444	0.0000	-0.1482	0.2593
0	3	0.3675	0.0199	-0.0816	0.2298	0.4444	0.0000	-0.1358	0.2541
0	4	0.3934	0.0124	-0.1030	0.2357	0.4444	0.0000	-0.1296	0.2523
0	5	0.4110	0.0084	-0.1193	0.2394	0.4444	0.0000	-0.1259	0.2515
0.4	1	0.2302	0.0587	-0.0441	0.2203	0.4603	-0.0335	-0.2282	0.3061
0.4	2	0.3065	0.0259	-0.0796	0.2306	0.4343	-0.0083	-0.1676	0.2641
0.4	3	0.3472	0.0142	-0.1061	0.2368	0.4259	-0.0037	-0.1475	0.2563
0.4	4	0.3724	0.0089	-0.1262	0.2405	0.4218	-0.0021	-0.1376	0.2535
0.4	5	0.3894	0.0060	-0.1418	0.2429	0.4193	-0.0013	-0.1316	0.2523

* Corner solution at these values – derivative not defined.

For the joint monopolist, $\frac{d\tilde{\Pi}_C}{dq_B}$ is also negative because the joint monopolist finds it optimal to raise the price of the base good sufficiently that it sacrifices some sales of the complementary good because some potential consumers find it too expensive to buy along with the base good. The results for $\frac{d\tilde{\Pi}_B}{dq_C}$ differ from that for independent firms. When $\tilde{\alpha} = 0$ base good profits are unaffected by changes in q_C because the joint monopolist has two price instruments and can adjust p_C without pricing any base-good only customers out of the market. When $\tilde{\alpha} > 0$, on the other hand, $\frac{d\tilde{\Pi}_B}{dq_C} < 0$ because the joint monopolist does not have two prices it can set independently. The two prices are linked through the additional network effects. Each base good customer cares about how many other consumers buy the base good (the additional network effects) and buyers of the complementary good must also buy the base good affecting its installed base. As a result, adjusting \tilde{p}_C affects x_B through $\tilde{\alpha}$.

Comparing columns 1 with 5 and 7 of Table 5, we see that increases in the quality of the base good have a smaller positive effect on the total (base and complementary good) normalized profits of the joint monopolist than on the normalized profits of the base good monopolist when there are independent monopolists. Thus, an independent base good monopolist has a greater marginal incentive to improve the base good than a joint monopolist. This is because the independent monopolist does not internalize the negative effect that a higher base good price has on the profits of the complementary good (as reflected in the negative values of $\frac{d\tilde{\Pi}_C}{dq_B}$ in column 3), while the joint monopolist does (as reflected in the negative values of $\frac{d\tilde{\Pi}_C}{dq_B}$ in column 7).

Comparing columns 4 with 6 and 8 of Table 5, we see that increases in the quality of the complementary good have a greater positive effect on the total (base and complementary good) normalized profits of the joint monopolist than on the normalized profits of the complementary good monopolist when there are independent monopolists. Thus, the joint monopolist has a greater marginal incentive to improve the complementary good than an independent monopolist selling the complementary good in the presence of an independent base good monopolist. This is because the joint monopolist can adjust the complementary good price fully (partially) to reflect the complementary good quality increase without affecting sales of the base good (as much) when $\tilde{\alpha} = 0$ ($\tilde{\alpha} > 0$). The independent complementary good monopolist, on the other hand, has to share some of the benefits of the complementary good improvement with the independent base good monopolist as reflected in the positive values of $\frac{d\tilde{\Pi}_B}{dq_C}$ in column 2.

Also note that $\frac{d\tilde{\Pi}_B}{dq_B} + \frac{d\tilde{\Pi}_C}{dq_B}$ is greater for the joint monopolist than for the independent firms which means that the effect on the joint monopolist's normalized profits from an increase in the base good quality is greater than the effect on the combined normalized profits of the independent firms. This is because the joint monopolist is better able to capture the benefits of increasing the base good quality by adjusting the complementary good price optimally.

We can also use our model to assess the marginal incentive to increase compatibility between the base good and complementary goods. Firms in markets with network effects, like software, often face decisions about the degree to which their product should be made compatible with other products or conform to industry standards. In our model this is equivalent to determining the effect on profits of an increase in normalized network effects ($\tilde{\alpha}$). We compare this incentive at different normalized quality levels and for different market structures (independent firms versus a joint monopolist).

First, we look at the effect of increasing additional network effects on the normalized profits of independent base good and complementary good monopolists. Values of $\frac{d\tilde{\Pi}_B}{d\alpha}$ are in column 1 of Table 6 and values of $\frac{d\tilde{\Pi}_C}{d\alpha}$ in column 2.²⁵ Second, we assess the effect of increasing normalized network effects on the normalized profits of the joint monopolist. Values of $\frac{d\tilde{\Pi}_B}{d\alpha}$ are in column 3 and values of $\frac{d\tilde{\Pi}_C}{d\alpha}$ are in column 4 of Table 6 and the effect on the total normalized profits of the joint monopolist is the sum of the two columns. Each row of Table 6 provides the effect at a given combination of normalized network effects and quality levels. For example, row two of the table indicates that at normalized complementary good quality of $\tilde{q}_C = 3$ and normalized network effects of $\tilde{\alpha} = 0.2$, a marginal increase in compatibility (normalized network effects) increases the normalized profits of the independent base good monopolist by 0.3644, the independent complementary good monopolist by 0.1853, the normalized base good profits of the joint monopolist by 0.3939 and the normalized complementary good profits of the joint monopolist by 0.1284.

Comparing the sum of columns 3 and 4 to columns 1 and 2 of Table 6, we observe that normalized profits are more sensitive to normalized network effects for a joint monopolist than for either independent monopolist. When additional network benefits are greater the value goes up both to consumers of the base good and to consumers of the complementary good. The value of the base good goes up directly because of greater availability of additional complementary goods. The value to complementary goods consumers goes up because they must buy the base good to use the complementary good and therefore also benefit indirectly from the increased availability of additional complementary goods. This is why $\frac{d\tilde{\Pi}_B}{d\alpha}$ and $\frac{d\tilde{\Pi}_C}{d\alpha}$ are both positive

²⁵ Derivatives are calculated using Richardson extrapolation with a step-size of one percent of $\tilde{\alpha}$.

for the independent monopolists (columns 1 and 2 of Table 6). In fact, the complementary good monopolist receives substantial benefits from the increase of network effects. However, because each of the independent firms does not take fully into consideration the effect they have on each other, their individual incentives to make their products more compatible are lower than the incentive of the joint monopolist to make its two products more compatible with other firms' goods. Since consumers of both the base and complementary goods benefit, the joint monopolist captures both benefits, while in the case of the independent monopolists this benefit is shared between the two firms.

Table 6 Effects of the Intensity of Additional Network Effects on Profits (“IF” = Independent Firms, “JM” = Joint Monopolist)

Normalized α $\left(\tilde{\alpha} = \frac{\alpha}{q_B}\right)$	Normalized Complementary Good Quality $\left(\tilde{q}_C = \frac{q_C}{q_B}\right)$	1	2	3	4
		IF $\frac{d\tilde{\Pi}_B}{d\alpha}$	IF $\frac{d\tilde{\Pi}_C}{d\alpha}$	JM $\frac{d\tilde{\Pi}_B}{d\alpha}$	JM $\frac{d\tilde{\Pi}_C}{d\alpha}$
0.1	3	0.3260	0.1776	0.3606	0.1214
0.2	3	0.3644	0.1853	0.3939	0.1284
0.3	3	0.4089	0.1900	0.4301	0.1352
0.4	3	0.4620	0.1892	0.4698	0.1415
0.5	3	0.5255	0.1807	0.5145	0.1461
0.6	3	0.6024	0.1615	0.5666	0.1470
0.7	3	0.6941	0.1300	0.6304	0.1412
0.8	3	0.7979	0.0879	0.7131	0.1231
0.9	3	0.9045	0.0416	0.8266	0.0831
0.1	5	0.3116	0.1999	0.3475	0.1319
0.2	5	0.3461	0.2098	0.3820	0.1354
0.3	5	0.3866	0.2170	0.4207	0.1380
0.4	5	0.4347	0.2199	0.4641	0.1393
0.5	5	0.4931	0.2158	0.5137	0.1386
0.6	5	0.5651	0.2009	0.5716	0.1343
0.7	5	0.6541	0.1714	0.6413	0.1242
0.8	5	0.7613	0.1249	0.7282	0.1042
0.9	5	0.8815	0.0644	0.8419	0.0673

6. Pricing of Windows and Office

One of the puzzles of the Microsoft antitrust case was the fact that Microsoft was charging a price for its Windows operating system that was significantly lower than most economic models predict. At the same time, Microsoft was selling the Microsoft Office suite of

applications²⁶ at a significantly higher price than Windows, even though Microsoft's market share was comparable in the Windows and Office markets.²⁷ At the time of the antitrust trial, Microsoft sold the majority of Windows and Office units through original equipment manufacturers (OEMs). While precise OEM price data is proprietary and difficult to obtain, estimates of the Office to Windows price ratio are in the range of 1.4 to 3.75.²⁸ Various explanations of the price difference have been offered, but none seemed to explain the low relative Windows price except for the possibility of very strong potential competition in the operating systems market.²⁹

Four main failing explanations have been offered. The first explanation is that Microsoft was keeping the price of Windows low to increase network effects, allowing it to possibly increase its price in the future. This explanation is unsatisfying given that Microsoft continued pricing Windows low even after it had gained a very high market share. A second possible explanation is that the existing installed base of Windows constrained Microsoft's pricing because consumers who bought a new computer would uninstall Windows from their old computer and install it on the new one. However, Microsoft's licensing requirements and the sheer complexity of uninstalling the operating system make it almost impossible for a user to uninstall a Windows operating system that was pre-installed by a computer hardware manufacturer and move it to a different (presumably new) computer. Moreover, typically, U.S. users who buy Windows pre-installed on their new computer are not given software that would allow them to install Windows to a *different* computer.³⁰ So it is unlikely that the Windows installed base constrained the Windows price. A third possible explanation is that since computer systems (hardware and software) are durable, pricing of new versions of Windows is constrained by the availability of old computer system versions (including Windows). However, very rapid technological change in hardware has prompted consumers to buy new computers much faster than traditional obsolescence rates would imply and Windows was only a small part of the price of a new personal computer. Thus, it is unlikely that durability was a significant factor constraining the price of Windows. A fourth possible explanation is that the price of Windows is constrained by the possibility of consumers pirating the software. Although pirating of both Microsoft Office and Windows would have the same effect, it is more difficult to pirate Windows. Therefore, piracy issues do not explain the price difference between Windows and Microsoft Office.

²⁶ Microsoft Office typically includes Word, a word processor; Excel, a spreadsheet; PowerPoint, a presentations tool; Outlook, a personal information management tool; and Access, a database.

²⁷ Note that the marginal cost of Windows and Office were both close to zero and approximately the same since neither were shipped with paper manuals and both were generally pre-loaded by OEMs on computers during this time.

²⁸ See for example, "Some Experts Blame Rising Software Prices on Microsoft," *PCWorld.com* (<http://www.cnn.com/TECH/computing/9901/11/microrise.idg/>).

²⁹ See Economides (2001).

³⁰ U.S. users are typically given a "recovery" CD that allows them to restore the particular computer model they own, including Windows, to the original condition when it was shipped from the factory. Such a CD is unable to install Windows on any other computer model.

Another possible explanation that has been proposed and dismissed in the context of pure monopoly models for Windows and Microsoft Office³¹ is that Microsoft kept the price of Windows low because this allowed Microsoft to charge more for complementary goods, such as Microsoft Office, that it produces. In the context of pure monopoly models for Windows and Microsoft Office, this explanation was insufficient to explain the very different prices charged for Windows and Office. In contrast, our model, in which a joint monopolist sets prices of the base and complementary goods in the presence or absence of additional network effects from other complementary goods, is able to explain the relative prices of Windows and Office.

We can apply our Model 4, with Windows as the base good and Microsoft Office as the complementary good.³² Using equations (20) and (21), the ratio of prices in Model 4 can be expressed analytically as

$$\tilde{R}_{CB} = \frac{p_C}{p_B} = \frac{3q_C}{4q_B} - \frac{1}{2}. \quad (24)$$

Ratios of the price of Office to the price of Windows reported during the Microsoft antitrust trial can be explained as an equilibrium of our model. Ratios of 1.4 to 3.75 for the price of Office relative to Windows reported in the Microsoft antitrust trial require $\frac{76}{30} \leq \frac{q_C}{q_B} \leq \frac{17}{3}$. The equilibrium of Model 4 also implies

$$\frac{x_C}{x_B} = \frac{3}{4} + \frac{q_B}{6q_C}, \quad (25)$$

from equations (20) and (21). We can use the actual relative sales ratio of Microsoft Office and Windows to infer the underlying relative qualities of the two goods and the equilibrium price ratio they imply. To determine the relative sales of Word and Office we obtained survey data from the Current Population Survey (CPS) Supplement on Computer and Internet Use from September 2001.³³ The survey asked the following questions about spreadsheet and word processors for both home and office use:

1. Do you use the computer at home (at the office) for word processing or desktop publishing?
2. Do you use the computer at home (at the office) for spreadsheets or databases?

We assumed that respondents answering “yes” to using a word processor either at work or at home used a word processor and similarly for spreadsheets. We then assumed that respondents using either a word processor or a spreadsheet (there were no questions relating to use of

³¹ See, for example, Economides (2001).

³² We present the case of zero additional network effects $\alpha = 0$ for brevity. The same result holds for positive network effects $\alpha > 0$ using Models 3 and 5.

³³ See <http://www.bls.census.gov/cps/> for details.

presentation or database software) owned an office suite.³⁴ Since only respondents who owned a computer answered either of these two questions, dividing the number of respondents we defined as owning an office suite would yield office suite owners as a fraction of operating system (OS) owners (since computers and operating systems are purchased in fixed proportions) or:

$$\frac{\text{office suite owners}}{\text{OS owners}} = \frac{\text{Microsoft Office} + \text{non-Microsoft Office owners}}{\text{Windows OS} + \text{non-Windows OS owners}}. \quad (26)$$

From the survey, we obtained a fraction of 0.82 for this ratio. Since,

$$\frac{x_C}{x_B} = \frac{\text{Microsoft Office owners}}{\text{Windows OS owners}} \quad (27)$$

we must adjust the numerator of equation (26) by Microsoft's share in the office suite market and the denominator of equation (26) by Microsoft's share in the operating system market so that we obtain:

$$\frac{x_C}{x_B} = 0.82 * \frac{\text{Microsoft Office share}}{\text{Windows OS share}}. \quad (28)$$

At the time of the antitrust trial, the market share of Windows among personal computers was estimated to be between 95% and 97%, while Microsoft Office's share among office suites was estimated to be 95%. This implies the ratio of Microsoft Office to Microsoft Windows sales in equation (28) is between 0.80 and 0.82. Using equation (25) this yields a normalized quality ratio of between 2.38 and 3.14 and, using equation (24) a price ratio of between 1.29 and 1.85, which is at the lower end of the range estimated during the trial.

It is interesting to compare the ratios of these prices under the two different market structures. Table 7 displays the normalized relative complementary good price obtained at given normalized quality levels for the joint monopolist (Model 4) versus independent firms (Model 2). Since Model 2 cannot be solved analytically we have displayed these results numerically. Except at low normalized quality levels of Office, a joint monopolist (such as Microsoft) has a higher normalized relative complementary good price than if an independent firm offered Office. Since the joint monopolist internalizes the complementary good profits, it prefers to keep the price of the base good low so as to not choke off the positive feedback with those complementary goods that Microsoft does not produce, while pricing the complementary good relatively high to benefit from sales to those with high demand for Office. If, on the other hand, an independent firm were to sell Office, Microsoft can only benefit from sales of those with high demand for Office by increasing the price of Windows. If the normalized quality of Office is sufficiently low, then it is more important for the joint monopolist to capture profits from the base good and it is priced relatively high, but for reasonably high normalized quality levels of the

³⁴ Although consumers could have purchased stand-alone word processor or spreadsheet software this was rare by 2001.

complementary good, the base good monopolist has an incentive to keep the base good price relatively low.

**Table 7 Ratio of Complementary to Base Good Prices:
Joint Monopolist Versus Independent Firms**

Normalized Complementary Good Quality $\left(\tilde{q}_C = \frac{q_C}{q_B}\right)$	Normalized Relative Price of Complementary Good $\left(\tilde{R}_{CB} = \frac{p_C}{p_B}\right)$	
	JM	IF
1	0.2500	0.7071
2	1.0000	1.2292
3	1.7500	1.7375
4	2.5000	2.2417
5	3.2500	2.7439
6	4.0000	3.2455
7	4.7500	3.7465
8	5.5000	4.2472
9	6.2500	4.7477
10	7.0000	5.2482

7. Relation to the Empirical Literature

In this section we demonstrate that our model can be used to simulate and calibrate results from the empirical literature estimating pricing effects of complementary goods and additional network effects. Our model could thus be used to estimate counterfactuals in these situations, sometimes with extensions to our model. Gandal, Kende and Rob (2000) estimate a structural model of adoption of CD players and complementary CD titles to determine the magnitude of network effects. Using data on the number of titles and CD players sold between 1985 and 1992, the authors find that the elasticity of the number of CD titles with respect to CD player sales is 0.56 while the elasticity of CD player sales with respect to the number of CD titles available is 0.033.

We can calibrate our Model 2 to these results. Model 2 is appropriate since firms selling CD players generally differed from those selling CD titles. We assume that the base good corresponds to CD players and the complementary good to CD titles. First, we find the equilibrium quantities of the base (x_B) and complementary (x_C) goods at given normalized quality level (\tilde{q}_C) and normalized utility intercepts³⁵ (which we define as $\tilde{k}_C = k_C/q_B$ and

³⁵ Model 2 is “scalable” in k_B, k_C, q_B and q_C in the sense that multiplying all four by a factor λ leaves sales of both goods unaffected and the normalized price unchanged.

$\tilde{k}_B = k_B/q_B$).³⁶ We then increase the complementary good normalized utility intercept to \tilde{k}_C' and find the equilibrium quantity of the complementary (x_C') and base goods (x_B') .

Increasing the complementary good normalized utility intercept simulates an increase in sales of the complementary good. We then compute the elasticity of base good sales with respect to complementary good sales: $\varepsilon_{BC} = (x_B' - x_B)x_C / ((x_C' - x_C)x_B)$. We simulate the elasticity of

complementary good sales with respect to base good sales in a similar manner by increasing the base good normalized utility intercept to \tilde{k}_B' (while holding \tilde{k}_C constant) and calculating

$$\varepsilon_{CB} = (x_C' - x_C)x_B / ((x_B' - x_B)x_C).$$

We find that at $\tilde{k}_B = 0.3$, $\tilde{k}_C = 0.1$, and $\tilde{q}_C = 2$ we get $\varepsilon_{BC} = 0.018$ and $\varepsilon_{CB} = 0.319$ which are close to the empirical results. At these values, $\tilde{R}_{CB} = 1.071$, normalized profits of the base good monopolist are 0.587 and normalized profits of the complementary good monopolist are 0.462. Once calibrated, various counterfactuals can be performed. For example, the sales, prices and profits of CD players and CD titles resulting from a merger of CD player and CD title manufacturers could be simulated by evaluating Model 4 at these parameter values. The effect of increased title variety could be examined by increasing $\tilde{\alpha}$ and solving for the new equilibrium, while increases in CD player quality (relative to CD quality) could be simulated by decreasing \tilde{q}_C and solving for the new equilibrium.

Gandal (1995) estimates a hedonic model of personal computer database management systems (DBMS) software pricing. Using data on all major products offered from 1989 to 1991, Gandal estimates the value of a DBMS being compatible with the Lotus spreadsheet, the dominant spreadsheet at the time. Compatibility with the Lotus standard meant that the DBMS could export files in a Lotus-compatible format. Gandal finds that DBMS products compatible with the Lotus standard had a 31% higher price relative to incompatible DBMS's, controlling for other quality variables.

We simulate this using our Model 2 (since the DBMS's and Lotus spreadsheet were produced by separate firms) and assume that the base good is a DBMS and the complementary good is the Lotus spreadsheet. First, we find the equilibrium normalized relative complementary price (\tilde{R}_{CB}) at a given normalized quality level (\tilde{q}_C). We then increase the normalized quality to \tilde{q}_C' and find the new equilibrium normalized relative complementary good price (\tilde{R}_{CB}') allowing the quantity to adjust optimally. Finally, we compute the elasticity of the complementary good price with respect to the change in complementary good quality:

³⁶ In this comparison, we reintroduce the possibility of positive k_B and k_C .

$\varepsilon_{pq} = \left(\tilde{R}_{CB}' - \tilde{R}_{CB} \right) \tilde{q}_C / \left(\left(\tilde{q}_C' - \tilde{q}_C \right) \tilde{R}_{CB} \right)$. We cannot calibrate our model to the empirical results in this case since we cannot measure the quality improvement equivalent to compatibility with the Lotus standard. As an example, however, at $\tilde{k}_B = \tilde{k}_C = 0$ and $\tilde{q}_C = 2$ we get $\varepsilon_{pq} = 1.075$.

Rysman (2004) uses a structural model to estimate the indirect network effects between consumer usage (measured by number of references per household per month from surveys) and quantity of advertisements in the yellow pages directory market. He finds that the elasticity of advertising with respect to consumer usage is 0.326 and the elasticity of consumer usage with respect to advertisement quantity is 0.154. Approximately 41% of the markets in Rysman's data set have a single publisher so that in theory we could calibrate our model in these monopoly markets. However, to do so would require a model of advertising demand to appropriately model the "price" of consumer advertising usage.

Brynjolfsson and Kemerer (1996) estimate a hedonic model of personal computer spreadsheet pricing on products sold between 1987 and 1992. The authors find that the elasticity of the spreadsheet price with respect to the size of the spreadsheet's installed base is 0.75. Since we do not explicitly model the dynamics of market share formation in our model, we cannot simulate this elasticity. However, a dynamic version of Model 2 with the spreadsheet product as the base good and a compatible product, such as the operating system, as the complementary good would allow simulation of these results.

Ohashi (2003) uses a random-coefficients discrete choice model to estimate the importance of indirect network effects in the standards battle between the Beta and VHS formats in the U.S. videocassette recorder market between 1978 and 1996. Ohashi uses the installed base of the hardware format as a proxy for the availability of software (movie titles) available for each format. The model predicts elasticities of market share with respect to installed base of 0.63 in 1978 increasing to 1.03 in 1980 and falling to 0.96 in 1981. As with Brynjolfsson and Kemerer we cannot simulate these elasticities since we do not explicitly model the dynamics of share formation. In addition, since Sony manufactured Beta-compatible machines and multiple firms manufactured VHS machines we would require a model of multiple base-good firms.

Park (2003) also estimates indirect network effects in the VCR market with the goal of quantifying the proportion of consumer value attributable to the network effect. Although unable to identify separate network effects for the two competing formats, VHS and Beta, because of unobserved consumer expectations, the author is able to estimate the response of one format's network "advantage" (relative strength of its network) to relative sales in each period. Since the author measures this network advantage in terms of the consumers' value function these results are not normalized in a way applicable to our model.

Dranove and Gandal (2003) test for indirect network effects in the DVD market and the extent to which pre-announcement of the competing DIVX technology slowed adoption of the DVD technology. The authors use studio commitments to film releases in the DVD technology weighted by the films' box office revenues and the percentage of top 100 box-office films released in the DVD technology to measure software availability for the hardware (DVD). Controlling for the effect of the DIVX technology introduction, the authors estimate an elasticity

of DVD-player sales with respect to studio support of 0.18 (although it is not significant) and a semi-elasticity with respect to availability of box-office hits of 4.71. Calibration of these results would also require a model of multiple firms since the DVD format was an open standard and multiple studios were deciding whether to issue films in the DVD format.

Nair, et. al. (2004) estimate indirect network effects in the market for personal digital assistants (PDAs). The authors develop a measure of software availability by measuring downloads of software for each of the two competing operating systems, Palm and Microsoft, from a popular website. Their main model yields an elasticity of market share with respect to software variety of 0.326 and an elasticity of the stock of software titles available on a format with respect to the installed base of PDAs of 0.613. Again, our model cannot be directly applied to these results since there are two competing base good (PDA) firms and many software (complementary) good providers.

8. Concluding Remarks

This paper bridges the gap between two approaches in the network effects literature, the micro and macro approaches. In the micro approach, network effects are calculated from the sales of complementary goods explicitly defined in the model. In the macro approach, network effects are assumed in the utility functions of consumers. We develop an equivalence between these two approaches. We solve a model with two goods, a base good and a complementary good whose use requires the base good, for two alternative industry structures, joint monopoly and two independent monopolists. We then find the appropriate parameter values for network effects in a macro model that produce the same equilibrium results as a micro model. We assess the effect of changes in the inherent quality of the base and complementary goods and equate them to increases in the intensity of network effects required to maintain the same base good profits. We also evaluate the incentive to invest in either the base or complementary good quality and product compatibility. Finally, we are able to provide an economically rational explanation of Microsoft's relative pricing of Windows and Office and demonstrate how our model can be calibrated to empirical network effects studies to perform counterfactuals.

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